CS 61A Spring 2023

Tree Recursion, Python Lists

Discussion 4: February 15, 2023

##Walkthrough Videos

Feel free to try these problems on the worksheet in discussion or on your own, and then come back to reference these walkthrough videos as you study.

> To see these videos, you should be logged into your berkeley.edu email.
>
> **Note**: We unfortunately only have walkthrough videos for some of the problems on this worksheet this semester, please use the problem names to find the corresponding videos.

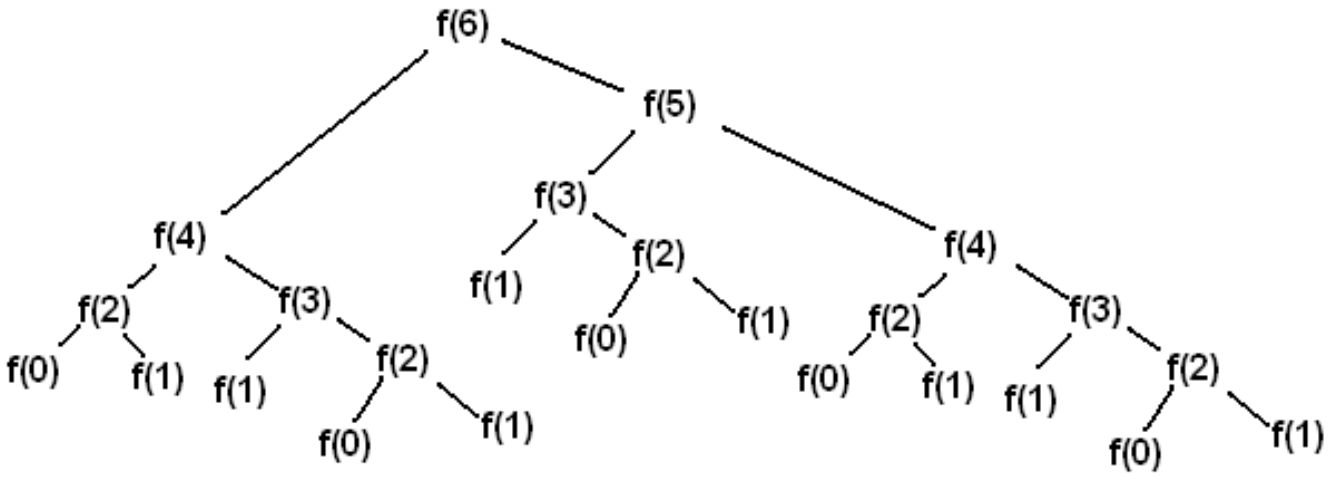
Tree Recursion

A tree recursive function is a recursive function that makes more than one call to itself, resulting in a tree-like series of calls.

For example, let's say we want to recursively calculate the `n`th Virahanka-Fibonacci number, defined as:

```
def virfib(n):
    if n == 0 or n == 1:
        return n
    return virfib(n - 1) + virfib(n - 2)
```

Calling `virfib(6)` results in the following call structure that looks like an upside-down tree (where `f` is `virfib`):



Virahanka-Fibonacci Tree

Each `f(i)` node represents a recursive call to `virfib`. Each recursive call `f(i)` makes another two recursive calls, which are to `f(i-1)` and `f(i-2)`. Whenever we reach a `f(0)` or `f(1)` node, we can directly return `0` or `1` rather than making more recursive calls, since these are our base cases.

In other words, base cases have the information needed to return an answer directly, without depending upon results

from other recursive calls. Once we've reached a base case, we can then begin returning back from the recursive calls that led us to the base case in the first place.

Generally, tree recursion can be effective for problems where there are multiple possibilities or choices at a current state. In these types of problems, you make a recursive call for each choice or for a group of choices.

Q1: Count Stair Ways

Imagine that you want to go up a flight of stairs that has `n` steps, where `n` is a positive integer. You can either take 1 or 2 steps each time. How many different ways can you go up this flight of stairs? In this question, you'll write a function `count_stair_ways` that solves this problem. Before you code your approach, consider these questions.

How many different ways are there to go up a flight of stairs with `n = 1` step? How about `n = 2` steps? Try writing out some other examples and see if you notice any patterns.

Solution: When there is only one step, there is only one way to go up the stair. When there are two steps, we can go up in two ways: take a single 2-step, or take two 1-steps.

What's the base case for this question? What is the simplest input?

Solution: Our first base case is when there is one step left. This is, by definition, the smallest input since it is the smallest positive integer. Our second base case is when we have two steps left. We need this base case for a similar reason that `fibonacci` needs 2 base cases: to cover both recursive calls.

Alternate solution: Our first base case is where there are no steps left. This means that we took an action in the previous recursive step that led to our goal of reaching the top. Our second base case is where we have overstepped. This means that the action we took is not valid, as it caused us to step over our goal.

What do `count_stair_ways(n - 1)` and `count_stair_ways(n - 2)` represent?

Solution: `count_stair_ways(n - 1)` represents the number of different ways to go up the last `n-1` stairs (this is the case where we take 1 step as our move). `count_stair_ways(n - 2)` represents the number of different ways to go up the last `n-2` stairs (this is the case where we take 2 steps as our move).

Fill in the code for `count_stair_ways`:

```
def count_stair_ways(n):
    """Returns the number of ways to climb up a flight of
    n stairs, moving either 1 step or 2 steps at a time.
    >>> count_stair_ways(4)
    5
    """
    if n == 1:
        return 1
    elif n == 2:
        return 2
    return count_stair_ways(n-1) + count_stair_ways(n-2)
```

Here's an alternate solution corresponding to the alternate base case presented above:

```
def count_stair_ways_alt(n):
    """Returns the number of ways to climb up a flight of
    n stairs, moving either 1 step or 2 steps at a time.
    >>> count_stair_ways_alt(4)
    5
    """
    if n == 0:
        return 1
    elif n < 0:
        return 0
    return count_stair_ways_alt(n-1) + count_stair_ways_alt(n-2)
```

You can use recursion visualizer to step through the calls made to `count_stair_ways(4)` for the original approach.

Q2: Count K

Consider a special version of the `count_stair_ways` problem, where instead of taking 1 or 2 steps, we are able to take up to and including `k` steps at a time. Write a function `count_k` that figures out the number of paths for this scenario. Assume `n` and `k` are positive.

Hint: Your solution will follow a very similar logic to what you did for `count_stair_ways`.

```
def count_k(n, k):
    """ Counts the number of paths up a flight of n stairs
    when taking up to and including k steps at a time.
    >>> count_k(3, 3) # 3, 2 + 1, 1 + 2, 1 + 1 + 1
    4
    >>> count_k(4, 4)
    8
    >>> count_k(10, 3)
    274
    >>> count_k(300, 1) # Only one step at a time
    1
    """
    if n == 0:
        return 1
    elif n < 0:
        return 0
    else:
        total = 0
        i = 1
        while i <= k:
            total += count_k(n - i, k)
            i += 1
        return total
```

We need to include the while loop from the `count_k` solution and keep track of a running total for the number of successful ways because we can take *up to* k steps. The while loop will count how many successful ways if we take 1, 2, 3, … k steps. We also need to keep track of how many successful ways there are for each value of k, so we use the `total` variable to remember how many successful ways there are so far.

You can use recursion visualizer to step through the calls made to `count_k(3, 3)`.

Lists

A list is a data structure that can store multiple elements. Each element can be of any type, even a list itself. We write a list as a comma-separated list of expressions in square brackets:

```
>>> list_of_ints = [1, 2, 3, 4]
>>> list_of_bools = [True, True, False, False]
>>> nested_lists = [1, [2, 3], [4, [5]]]
```

Each element in the list has an index, with the index of the first element starting at `0`. We say that lists are therefore "zero-indexed."

With list indexing, we can specify the index of the element we want to retrieve. A negative index represents starting from the end of the list, so the negative index `-i` is equivalent to the positive index `len(lst)-i`.

```
>>> lst = [6, 5, 4, 3, 2, 1, 0]
>>> lst[0]
6
>>> lst[3]
3
>>> lst[-1] # Same as lst[6]
0
```

List slicing

To create a copy of part or all of a list, we can use list slicing. The syntax to slice a list `lst` is: `lst[<start index>:<end index>:<step size>]`.

This expression evaluates to a new list containing the elements of `lst`:

- Starting at and including the element at `<start index>`.
- Up to but not including the element at `<end index>`.
- With `<step size>` as the difference between indices of elements to include.

If the start, end, or step size are not explicitly specified, Python has default values for them. A negative step size indicates that we are stepping backwards through a list when including elements.

```
>>> lst[:3]   # Start index defaults to 0
[6, 5, 4]
>>> lst[3:]   # End index defaults to len(lst)
[3, 2, 1, 0]
>>> lst[::-1]   # Make a reversed copy of the entire list
[0, 1, 2, 3, 4, 5, 6]
>>> lst[::2]  # Skip every other; step size defaults to 1 otherwise
[6, 4, 2, 0]
```

List comprehensions

List comprehensions are a compact and powerful way of creating new lists out of sequences. The general syntax for a list comprehension is the following:

```
[<expression> for <element> in <sequence> if <conditional>]
```

where the `if <conditional>` section is optional.

The syntax is designed to read like English: "Compute the expression for each element in the sequence (if the conditional is true for that element)."

```
>>> [i**2 for i in [1, 2, 3, 4] if i % 2 == 0]
[4, 16]
```

This list comprehension will:

- Compute the expression `i**2`
- For each element `i` in the sequence `[1, 2, 3, 4]`
- Where `i % 2 == 0` (`i` is an even number),

and then put the resulting values of the expressions into a new list.

In other words, this list comprehension will create a new list that contains the square of every even element of the original list `[1, 2, 3, 4]`.

We can also rewrite a list comprehension as an equivalent `for` statement, such as for the example above:

```
>>> lst = []
>>> for i in [1, 2, 3, 4]:
...     if i % 2 == 0:
...         lst = lst + [i**2]
>>> lst
[4, 16]
```

Q3: WWPD: Lists

What would Python display?

```
>>> a = [1, 5, 4, [2, 3], 3]
>>> print(a[0], a[-1])
```

1 3

```
>>>len(a)
```

5

```
>>> 2 in a
```

False

```
>>> a[3][0]
```

2

Video walkthrough

Q4: Even weighted

Write a function that takes a list `s` and returns a new list that keeps only the even-indexed elements of `s` and multiplies them by their corresponding index. First approach this problem with a normal `for` loop (without list comprehension).

```
def even_weighted_loop(s):
    """
    >>> x = [1, 2, 3, 4, 5, 6]
    >>> even_weighted_loop(x)
    [0, 6, 20]
    """
    result = []
    for i in range(len(s)):
        if i % 2 == 0:
            result = result + [i * s[i]]
    return result
```

Now that you've done it with a `for` loop, try it with a list comprehension!

```
def even_weighted_comprehension(s):
    """
    >>> x = [1, 2, 3, 4, 5, 6]
    >>> even_weighted_comprehension(x)
    [0, 6, 20]
    """
    return [i * s[i] for i in range(len(s)) if i % 2 == 0]
```

The key point to note is that instead of iterating over each element in the list, we must instead iterate over the indices of the list. Otherwise, there's no way to tell if we should keep a given element.

Q5: Max Product

Write a function that takes in a list and returns the maximum product that can be formed using nonconsecutive elements of the list. The input list will contain only numbers greater than or equal to 1.

```
def max_product(s):
    """Return the maximum product that can be formed using
    non-consecutive elements of s.
    >>> max_product([10,3,1,9,2]) # 10 * 9
    90
    >>> max_product([5,10,5,10,5]) # 5 * 5 * 5
    125
    >>> max_product([])
    1
    """
    if s == []:
        return 1
    else:
        return max(max_product(s[1:]), s[0] * max_product(s[2:]))
```

At each step, we choose if we want to include the current number in our product or not:

- If we include the current number, we cannot use the adjacent number.
- If we don't use the current number, we try the adjacent number (and obviously ignore the current number).

The recursive calls represent these two alternate realities. Finally, we pick the one that gives us the largest product.

Dictionaries

Dictionaries are data structures which map keys to values. Dictionaries in Python are unordered, unlike real-world dictionaries — in other words, key-value pairs are not arranged in the dictionary in any particular order. Let's look at an example:

```
>>> pokemon = {'pikachu': 25, 'dragonair': 148, 'mew': 151}
>>> pokemon['pikachu']
25
>>> pokemon['jolteon'] = 135
>>> pokemon
{'jolteon': 135, 'pikachu': 25, 'dragonair': 148, 'mew': 151}
>>> pokemon['ditto'] = 25
>>> pokemon
{'jolteon': 135, 'pikachu': 25, 'dragonair': 148,
'ditto': 25, 'mew': 151}
```

The *keys* of a dictionary can be any *immutable* value, such as numbers, strings, and tuples.[1] Dictionaries themselves are mutable; we can add, remove, and change entries after creation. There is only one value per key, however — if we assign a new value to the same key, it overrides any previous value which might have existed.

To access the value of `dictionary` at `key`, use the syntax `dictionary[key]`.

Element selection and reassignment work similarly to sequences, except the square brackets contain the key, not an index.

[1]To be exact, keys must be *hashable*, which is out of scope for this course. This means that some mutable objects, such as classes, can be used as dictionary keys.

Q6: WWPD: Dictionaries

What would Python display?

```
>>> pokemon = {'pikachu': 25, 'dragonair': 148}
>>> pokemon
```

{'pikachu': 25, 'dragonair': 148}

```
>>> 'mewtwo' in pokemon
```

False

```
>>> len(pokemon)
```

2

```
>>> pokemon['mew'] = pokemon['pikachu']
>>> pokemon[25] = 'pikachu'
>>> pokemon
```

{'pikachu': 25, 'dragonair': 148, 'mew': 25, 25: 'pikachu'}

```
>>> pokemon['mewtwo'] = pokemon['mew'] * 2
>>> pokemon
```

{'pikachu': 25, 'dragonair': 148, 'mew': 25, 25: 'pikachu', 'mewtwo': 50}

```
>>> pokemon[['firetype', 'flying']] = 146
```

Error: unhashable type

Note that the last example demonstrates that dictionaries cannot use other mutable data structures as keys. However, dictionaries can be arbitrarily deep, meaning the *values* of a dictionary can be themselves dictionaries.