Trees

```
def tree(root_label, branches=[]):
    for branch in branches:
        assert is_tree(branch), 'branches must be trees'
        return [root_label] + list(branches)

def label(tree):
    return tree[0]

def branches(tree):
    return tree[1:]

def is_leaf(tree):
    return not(branches(tree))
```

Trees

```
>>> t = tree(3, [tree(1), tree(2, [tree(4), tree(5)])])
>>> t -> [3, [1], [2, [4], [5]]]

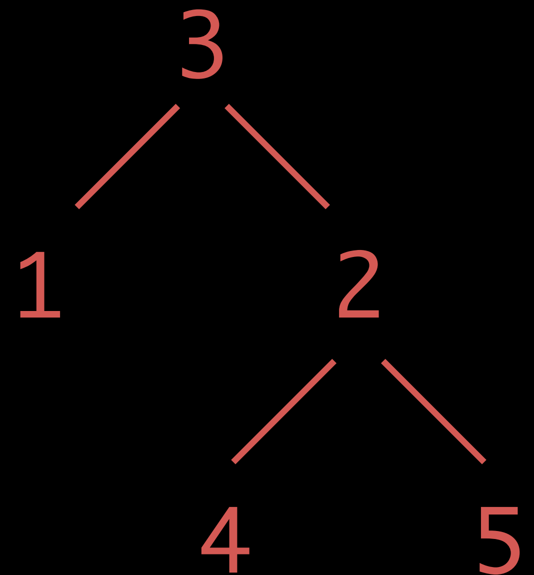
>>> label(t) -> 3

>>> branches(t) -> [[1], [2, [4], [5]]]

>>> label(branches(t)[0]) -> 1

>>> label(branches(t)[1]) -> 2

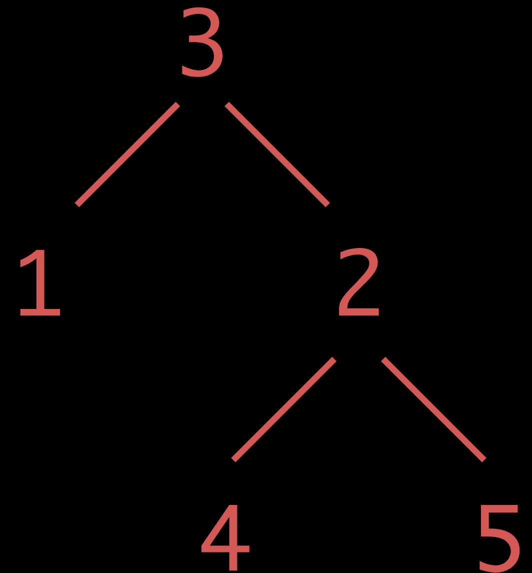
>>> r = branches(t)[1]
>>> label(branches(r)[0]) -> 4
>>> label(branches(r)[1]) -> 5
```



Trees (with objects)

```
1  # Trees with objects (manual tree construction)
2
3  class Tree:
4      def __init__(self, label, branches=[]):
5          self.label = label
6          self.branches = branches
7
8      def __repr__(self):
9          if self.branches:
10             return 'T[{0}, {1}]'.format(self.label, repr(self.branches))
11         else:
12             return 'T[{0}]'.format(repr(self.label))
13
14 t = Tree(3)
15 t.branches = [Tree(1),Tree(2)]
16 t.branches[1].branches = [Tree(4),Tree(5)]
17 print(t)
```

```
T[3, [T[1], T[2, [T[4], T[5]]]]]
```



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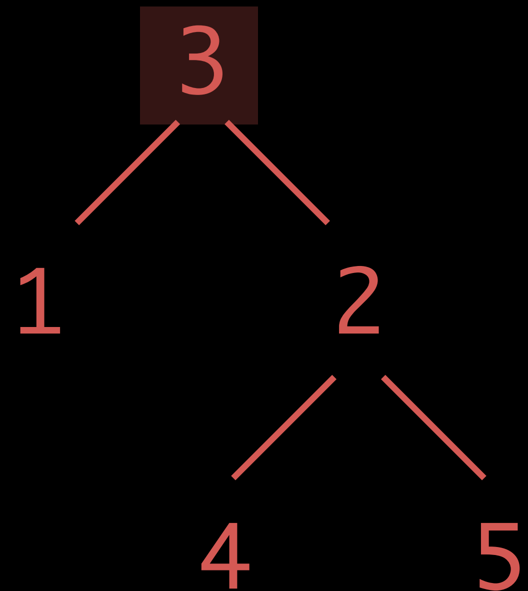
```

```
T[3, [T[1], T[2, [T[4], T[5]]]]]
```



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```

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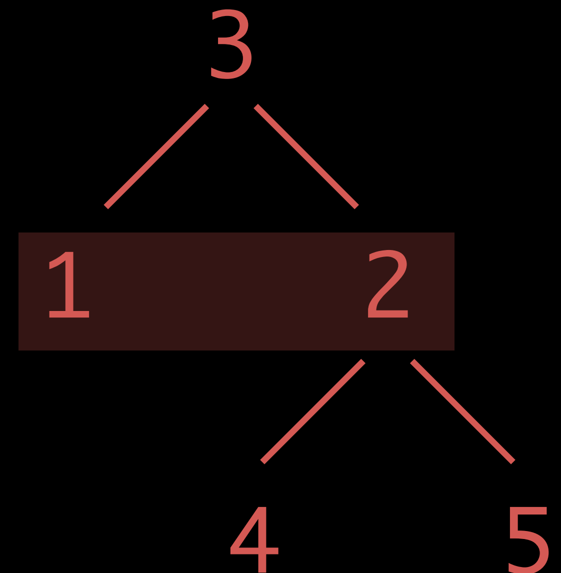

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```

```
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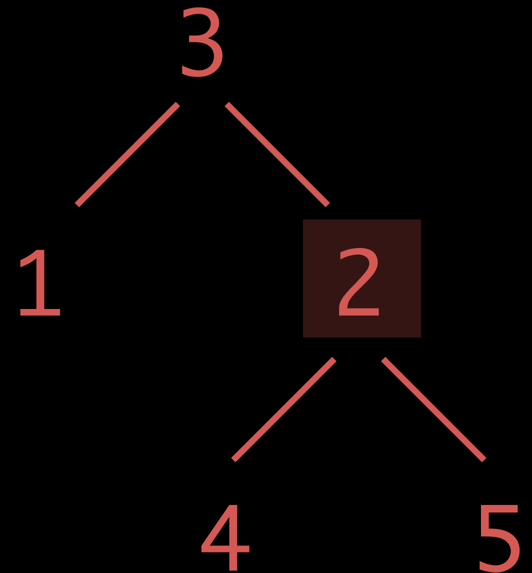

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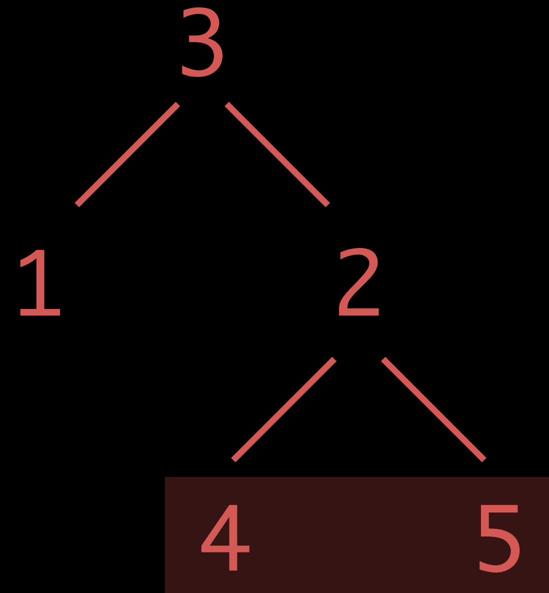

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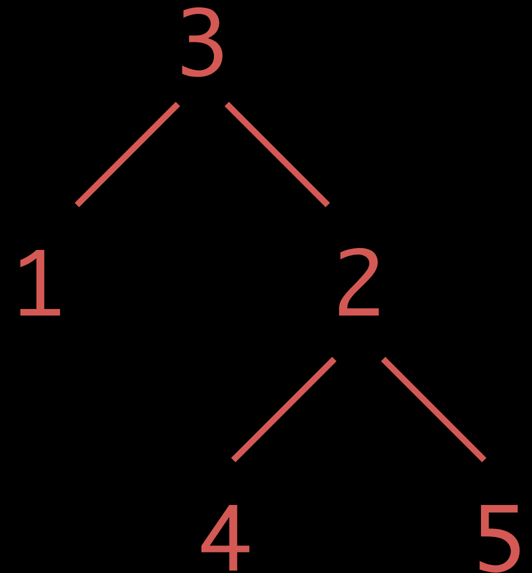
```

```
T[3, [T[1], T[2, [T[4], T[5]]]]]
```

poor data abstraction



Trees (with objects)

```
1  # Trees with objects (functional tree construction)
2
3  class Tree:
4      def __init__(self, label):
5          self.label = label
6          self.branches = []
7
8      # add child in the right-most branch
9      def add_child(self,val):
10         if not self.branches:
11             self.branches = [Tree(val)]
12         else:
13             self.branches.append(Tree(val))
14
15     # return subtree in location num (0-indexed)
16     def get_subtree(self,num):
17         # make sure that there is a child numbered num
18         if self.branches and num < len(self.branches):
19             return self.branches[num]
20         else:
21             return None
22
23 t = Tree(3)
24 t.add_child(1)
25 t.add_child(2)
26 c = t.get_subtree(1)
27 print(c)
28 c.add_child(4)
29 c.add_child(5)
30 print(t)
```

```
T[2]
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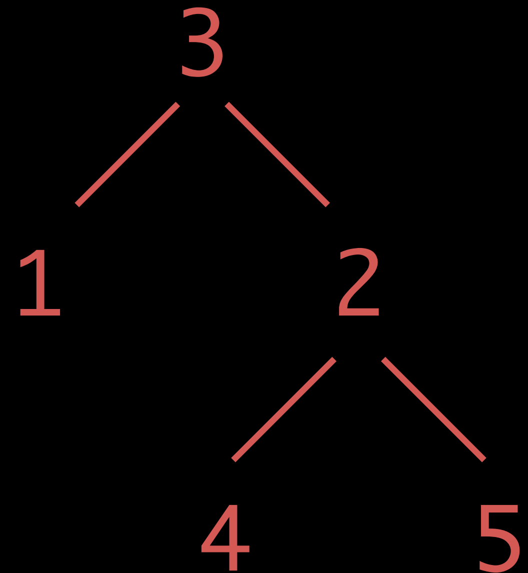
```



Trees (with objects)

```python
# Trees with objects (functional tree construction)

class Tree:
    def __init__(self, label):
        self.label = label
        self.branches = []

    # add child in the right-most branch
    def add_child(self,val):
        if not self.branches:
            self.branches = [Tree(val)]
        else:
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c = t.get_subtree(1)
print(c)
c.add_child(4)
c.add_child(5)
print(t)
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```
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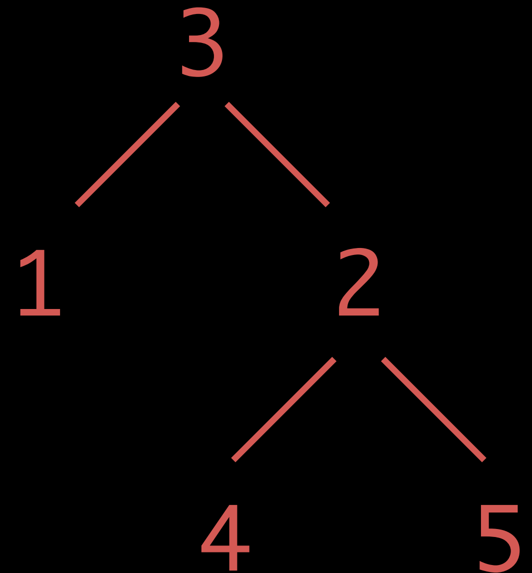

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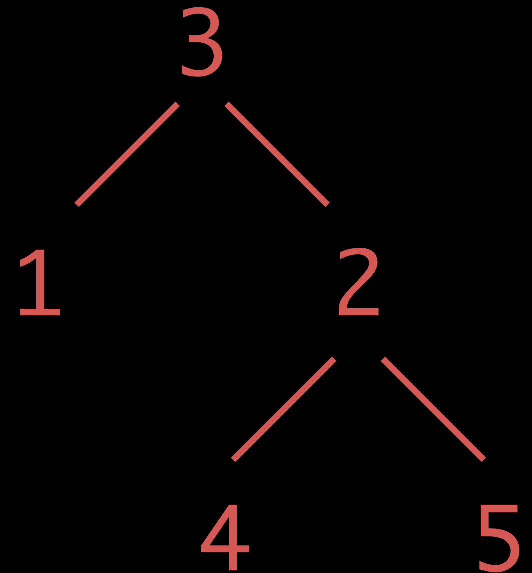

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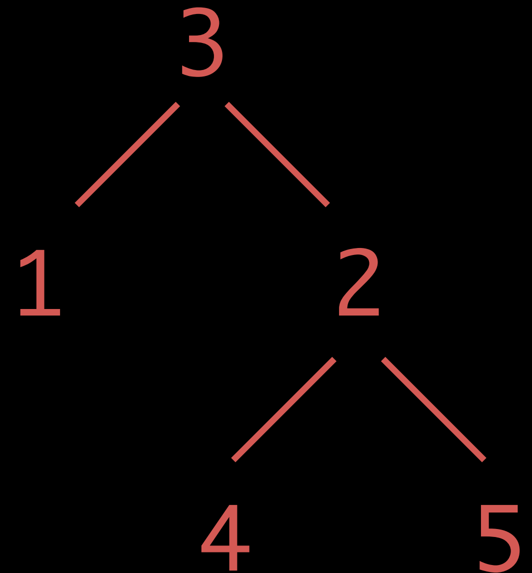

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```
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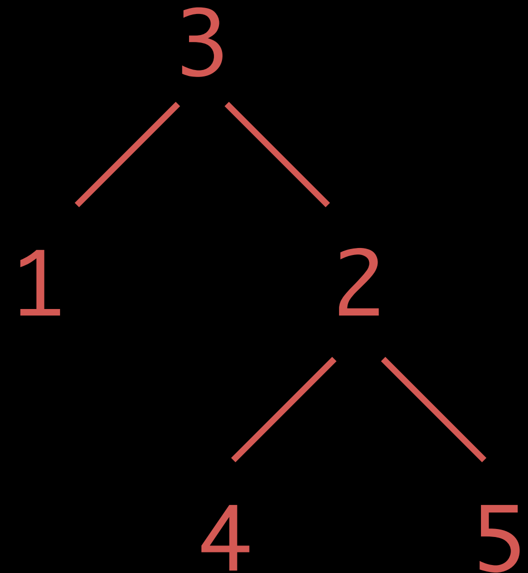
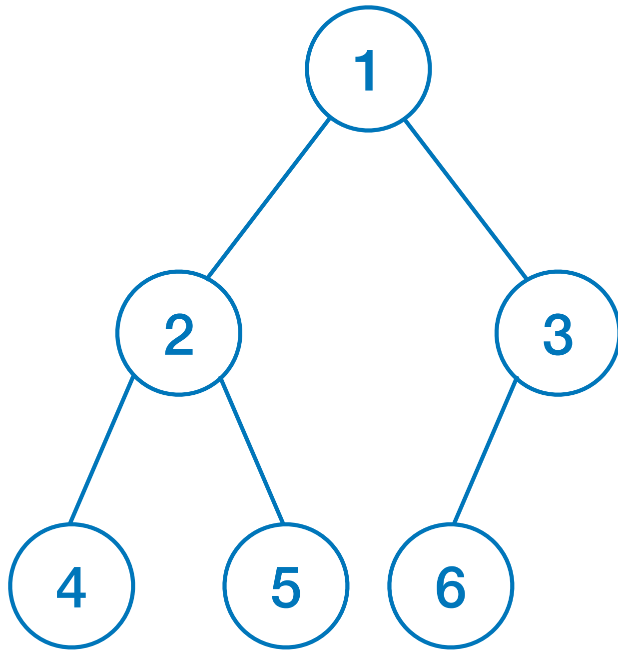


Binary Trees



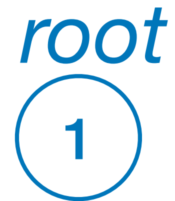
Binary Trees (representation)

```
1  # Binary Tree in Python
2  class Node:
3      def __init__(self, data):
4          self.label = data
5          self.left  = None
6          self.right = None
```

Binary Trees (representation)

```python
# Binary Tree in Python
class Node:
    def __init__(self, data):
        self.label = data
        self.left  = None
        self.right = None
```

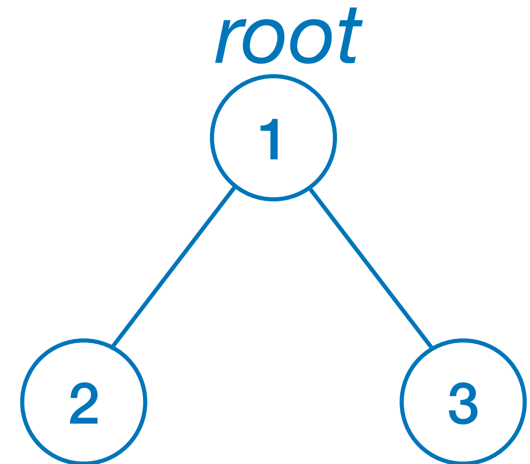
```python
root                    = Node(1)
```



Binary Trees (representation)

```python
# Binary Tree in Python
class Node:
    def __init__(self, data):
        self.label = data
        self.left  = None
        self.right = None
```

```python
root            = Node(1)
root.left       = Node(2)
root.right      = Node(3)
```



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# Binary Tree in Python
class Node:
    def __init__(self, data):
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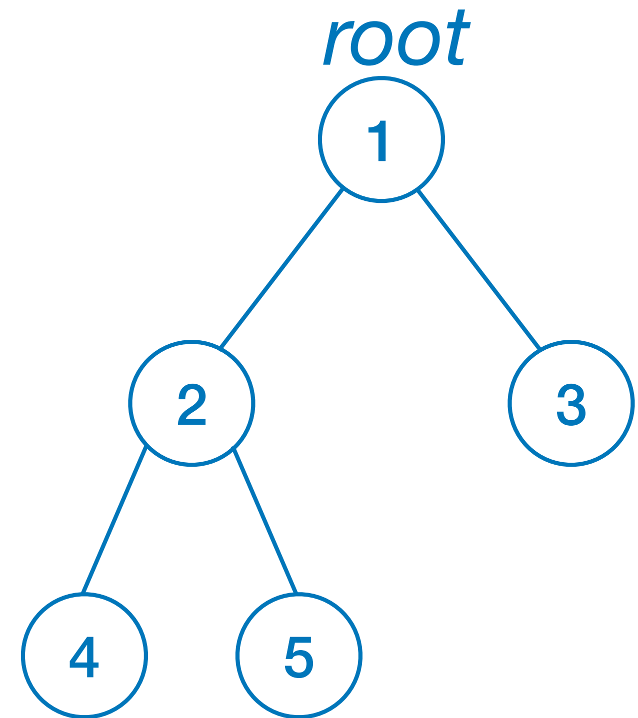
```

```python
root                = Node(1)
root.left           = Node(2)
root.right          = Node(3)
root.left.left      = Node(4)
root.left.right     = Node(5)
```



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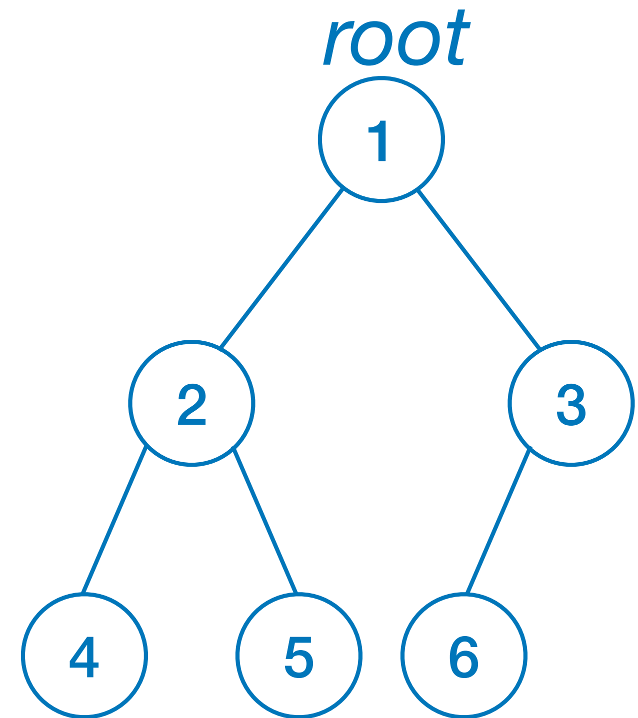
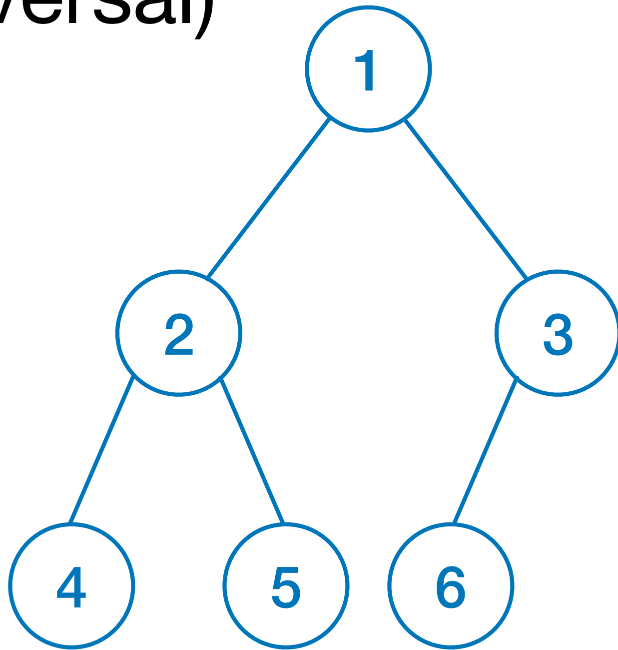
2  class Node:
3      def __init__(self, data):
4          self.label = data
5          self.left  = None
6          self.right = None
```

```
1  root              = Node(1)
2  root.left         = Node(2)
3  root.right        = Node(3)
4  root.left.left    = Node(4)
5  root.left.right   = Node(5)
6  root.right.left   = Node(6)
```

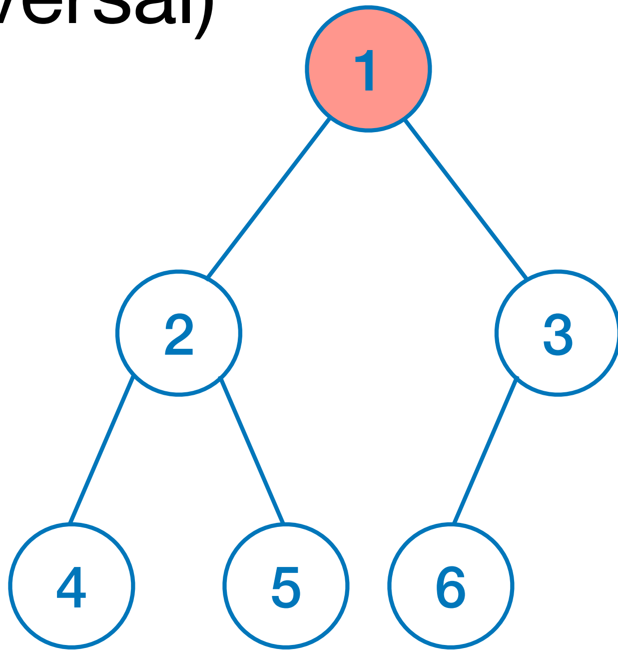


Binary Trees
(traversal)



preorder: root, left subtree, right subtree

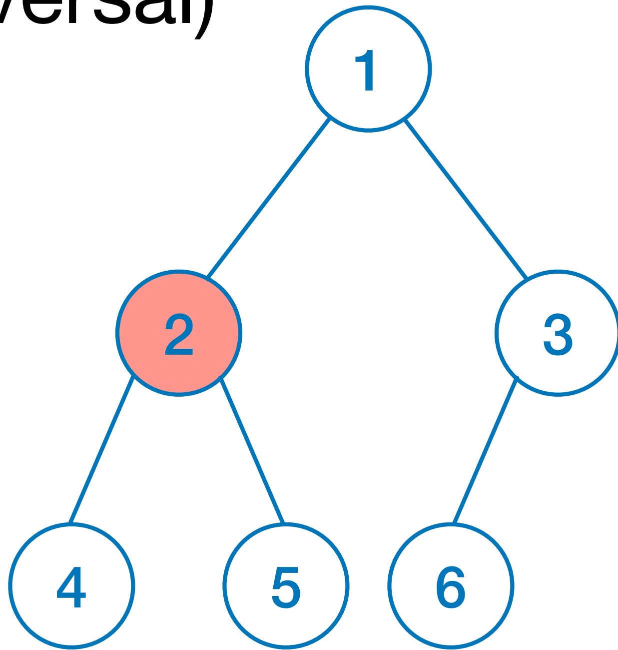
Binary Trees
(traversal)



preorder: root, left subtree, right subtree

1

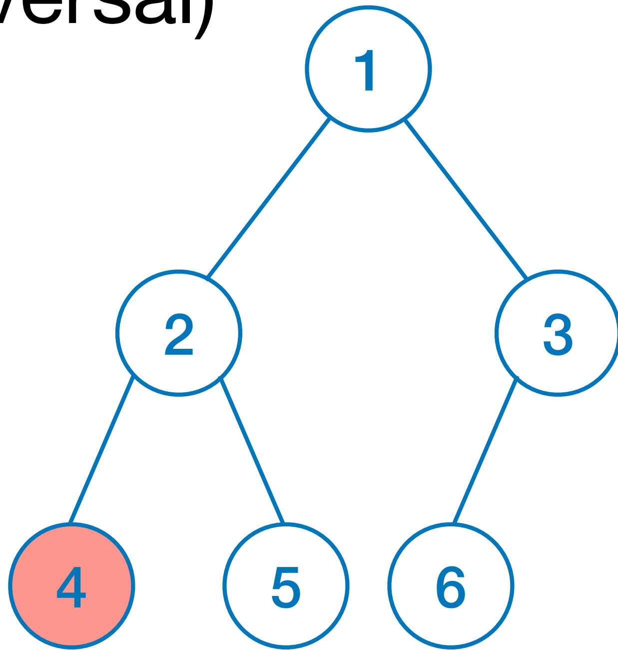
Binary Trees
(traversal)



preorder: root, left subtree, right subtree

1 2

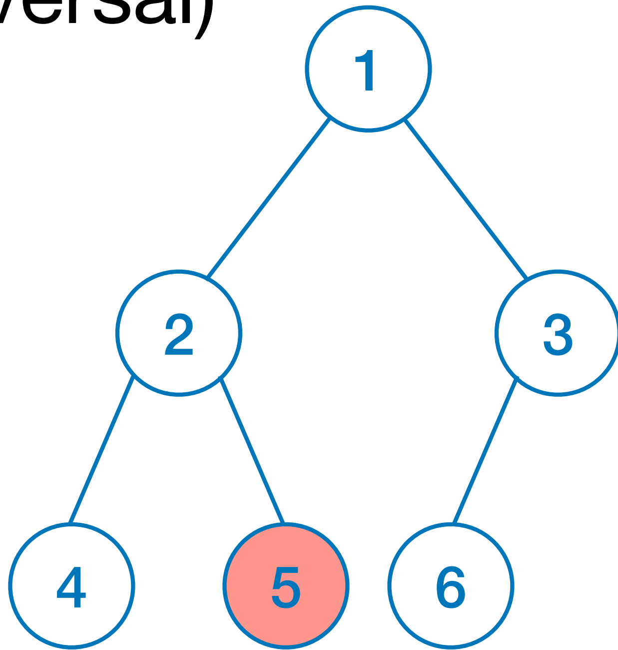
Binary Trees (traversal)



preorder: root, left subtree, right subtree

1 2 4

Binary Trees
(traversal)



preorder: root, left subtree, right subtree
1 2 4 5

Binary Trees
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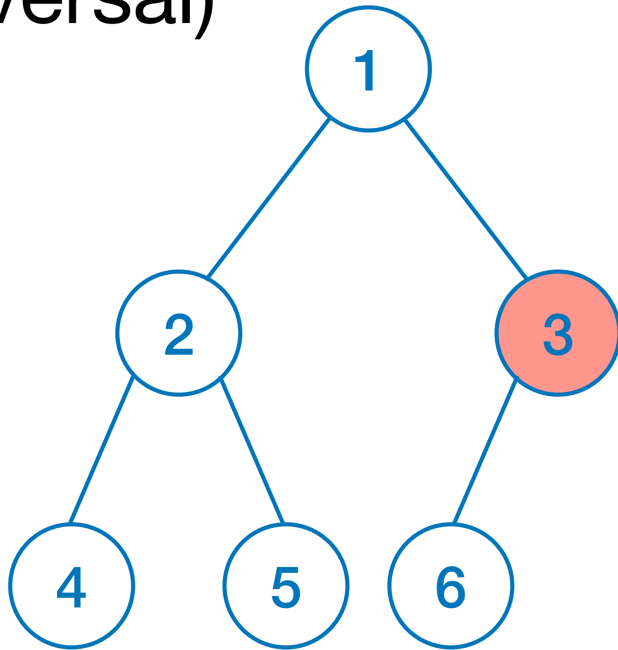
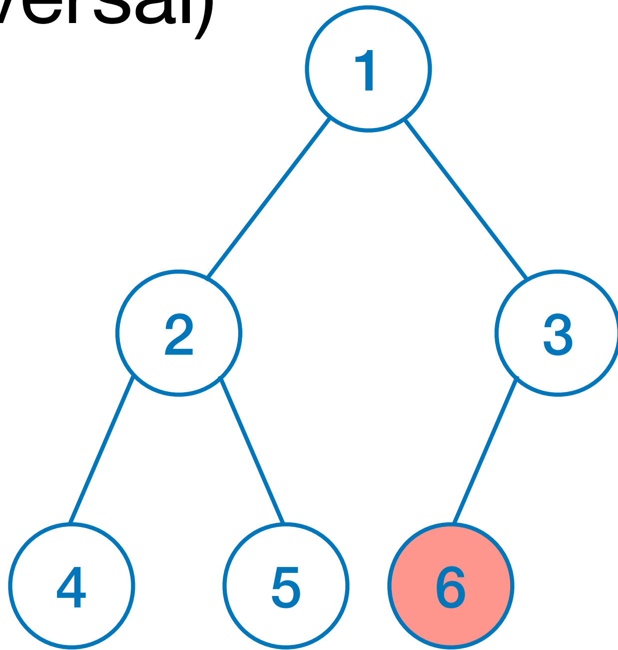


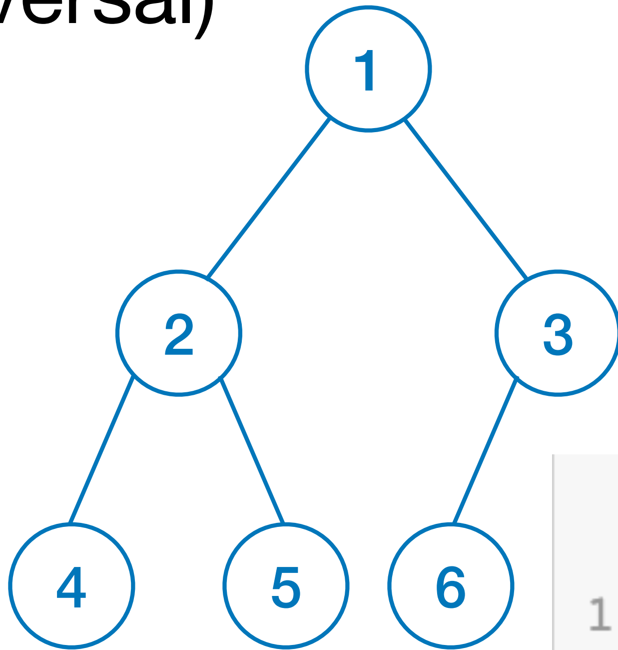
preorder: root, left subtree, right subtree
1 2 4 5 3

Binary Trees
(traversal)



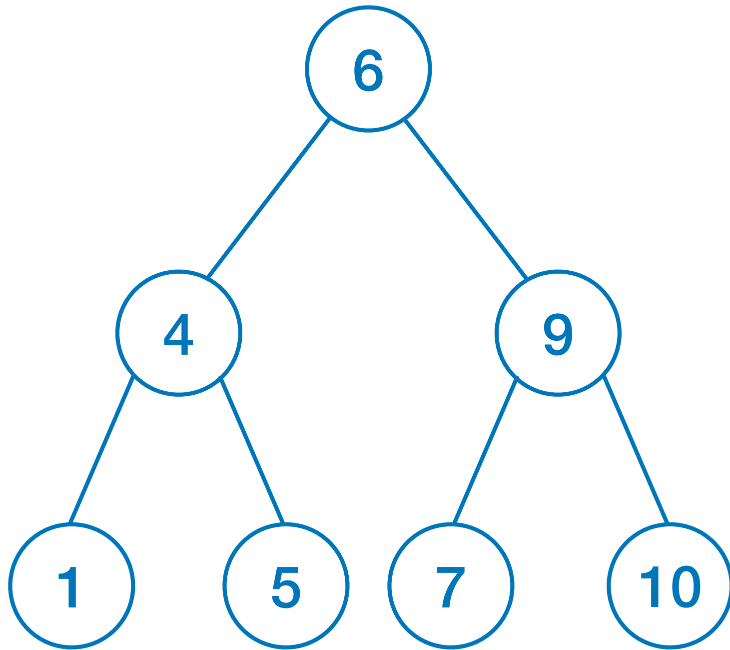
preorder: root, left subtree, right subtree
1 2 4 5 3 6

Binary Trees (traversal)

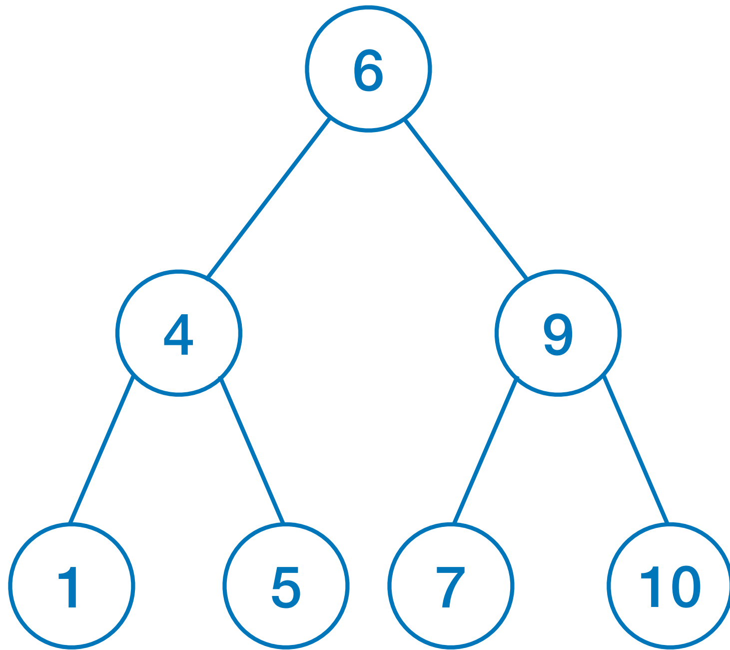



```
 8     # Traverse preorder
 9     def traversePreOrder(self):
10         print(self.label, end=' ')
11         if self.left:
12             self.left.traversePreOrder()
13         if self.right:
14             self.right.traversePreOrder()
```

Binary Search Trees

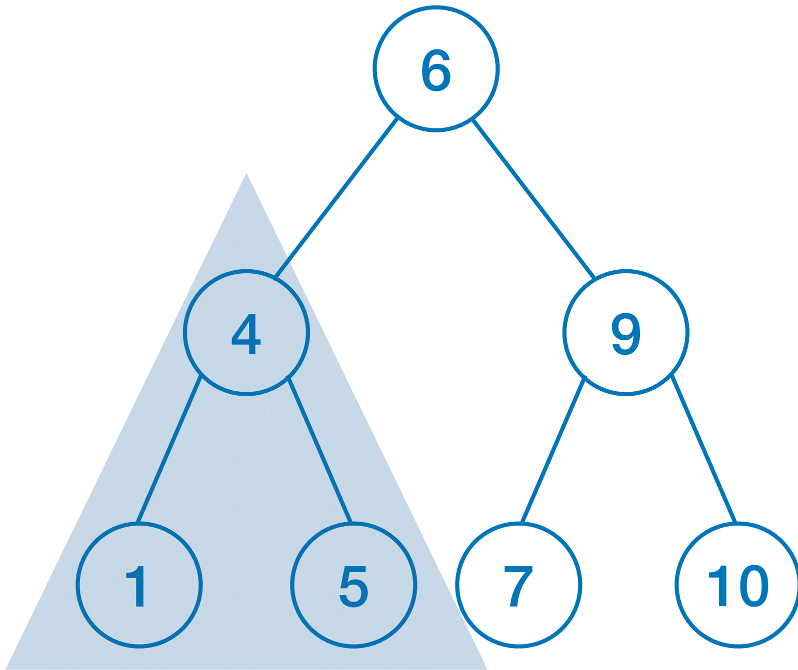


Binary Search Trees



all nodes in left subtree are less than root
all nodes in right subtree are larger than root

Binary Search Trees



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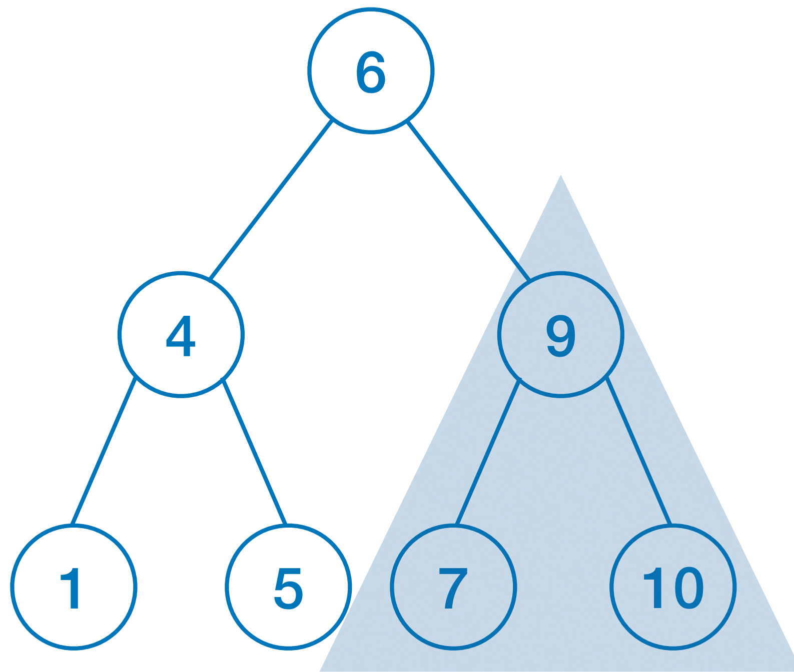

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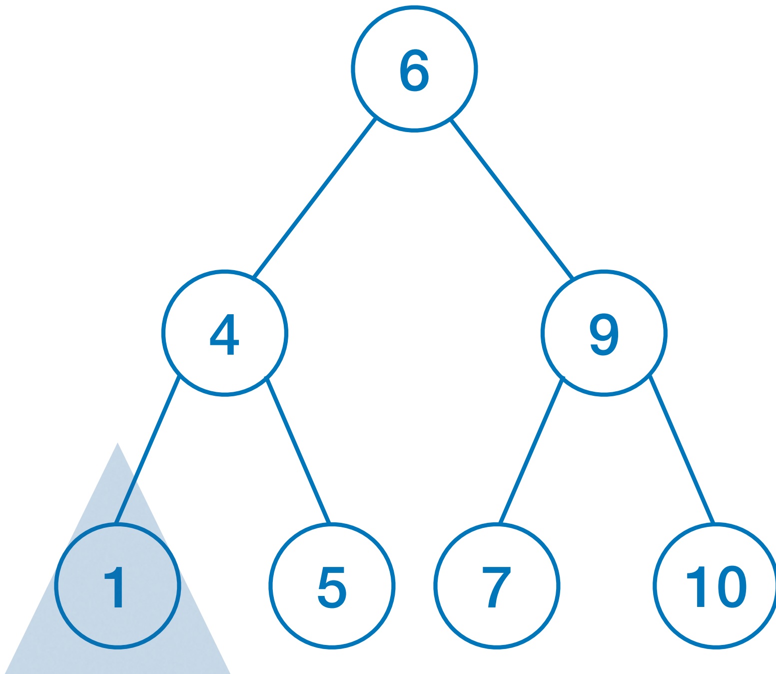
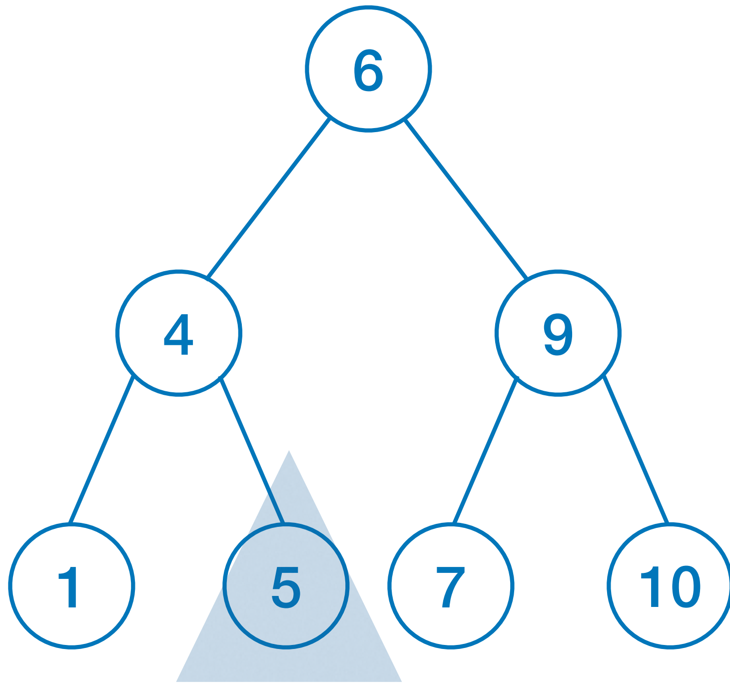
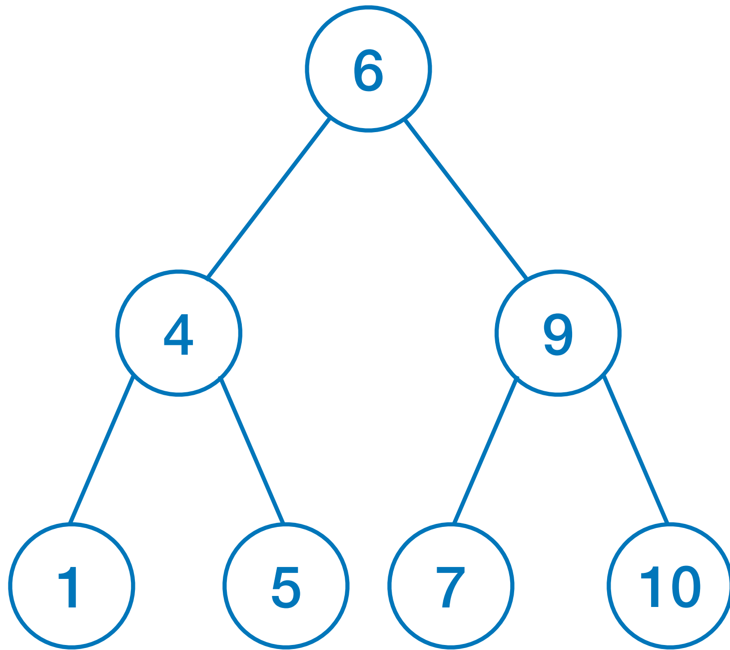


Binary Search Trees

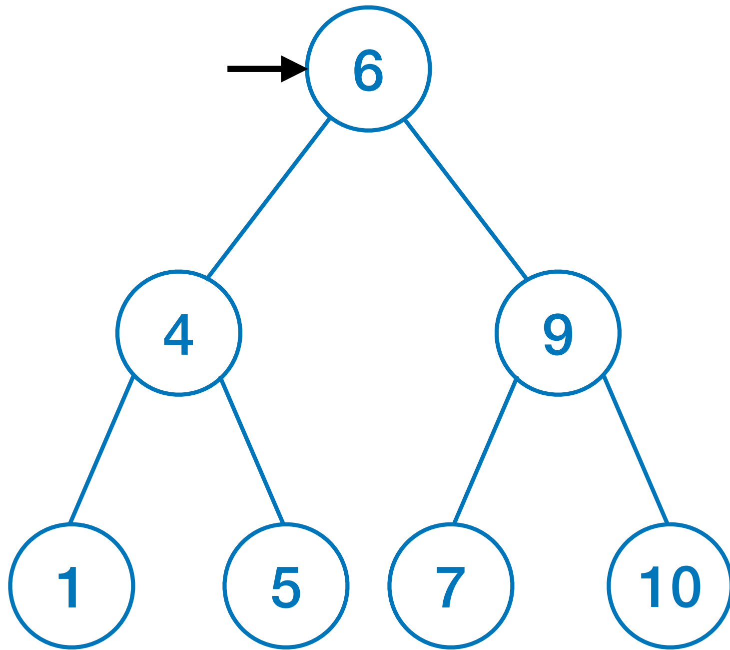


Binary Search Trees



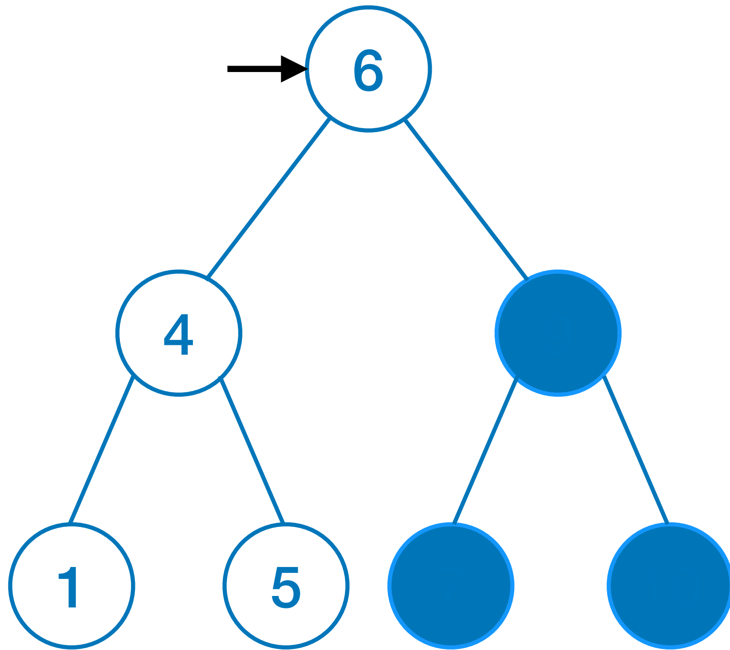
is 5 in this tree?
search in preorder (root, left, right)
n comparisons

Binary Search Trees



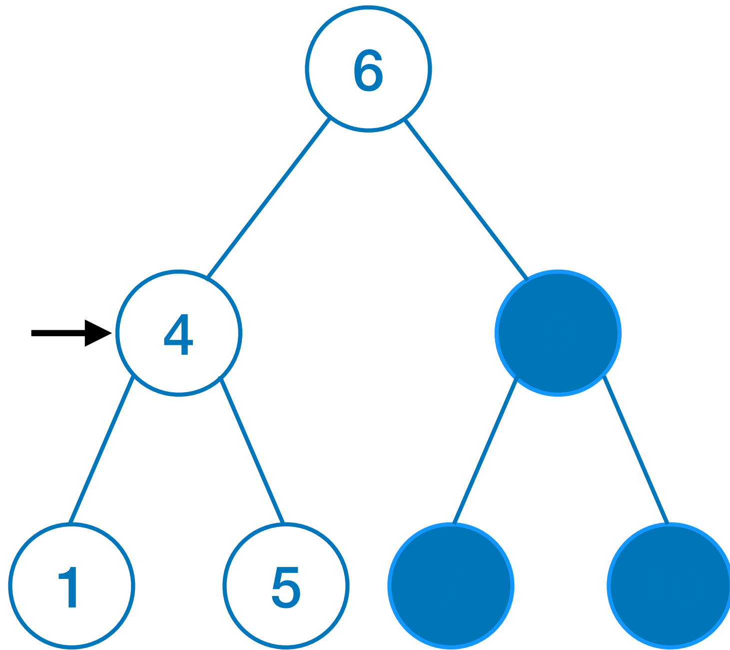
is 5 in this tree?

Binary Search Trees



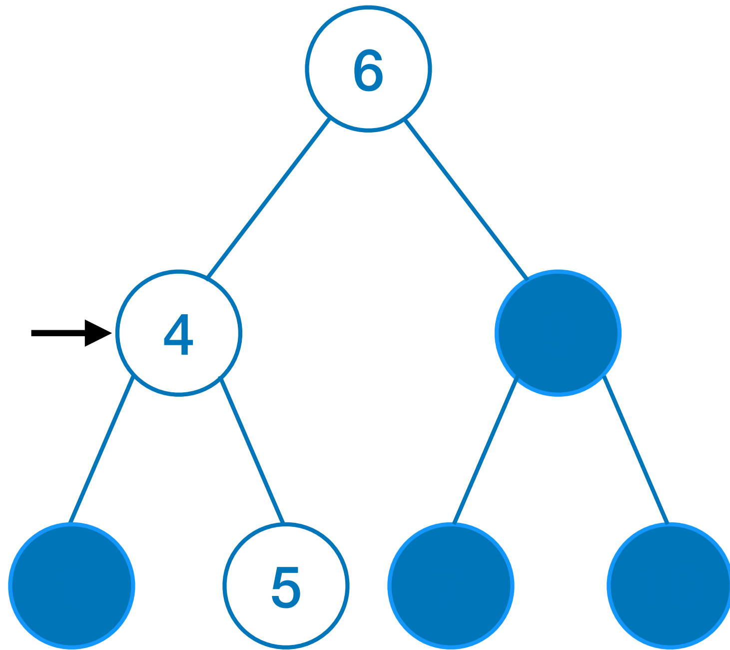
is 5 in this tree?
5 cannot be in right subtree

Binary Search Trees



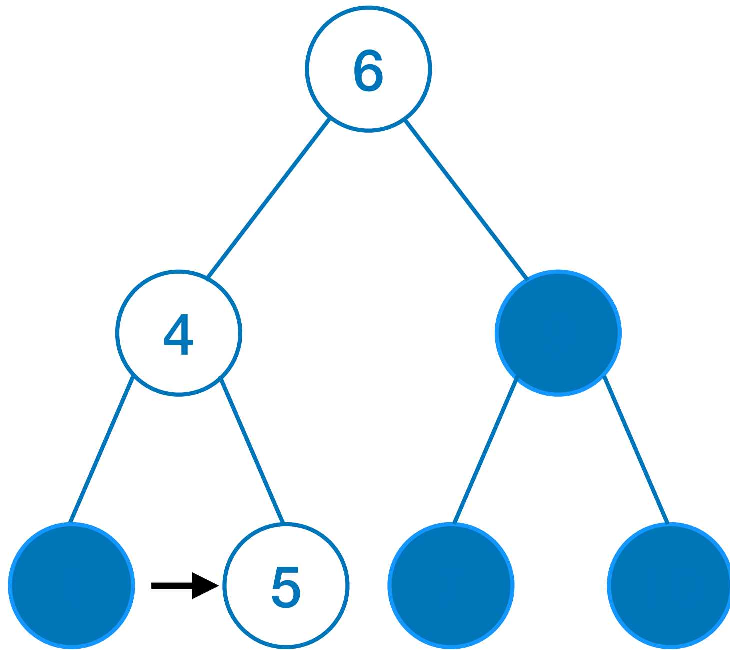
is 5 in this subtree?

Binary Search Trees



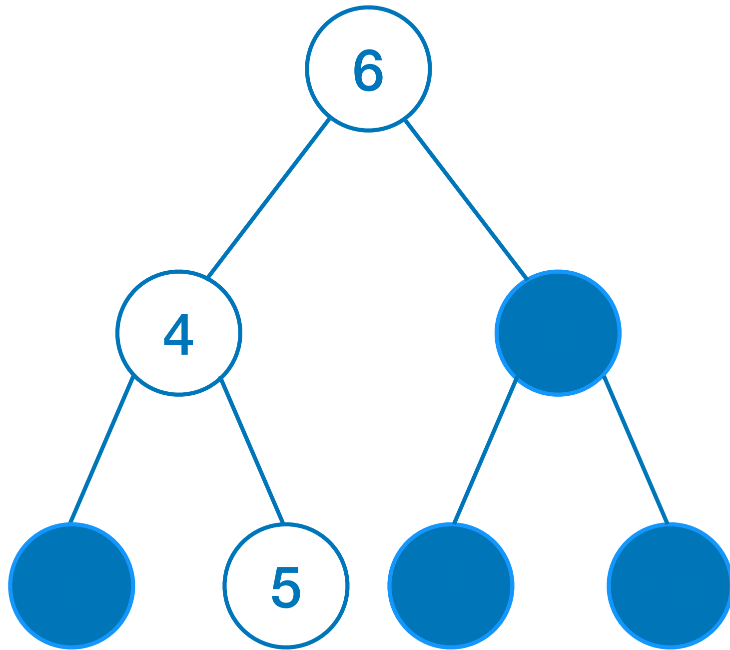
is 5 in this subtree?
5 cannot be in left subtree

Binary Search Trees



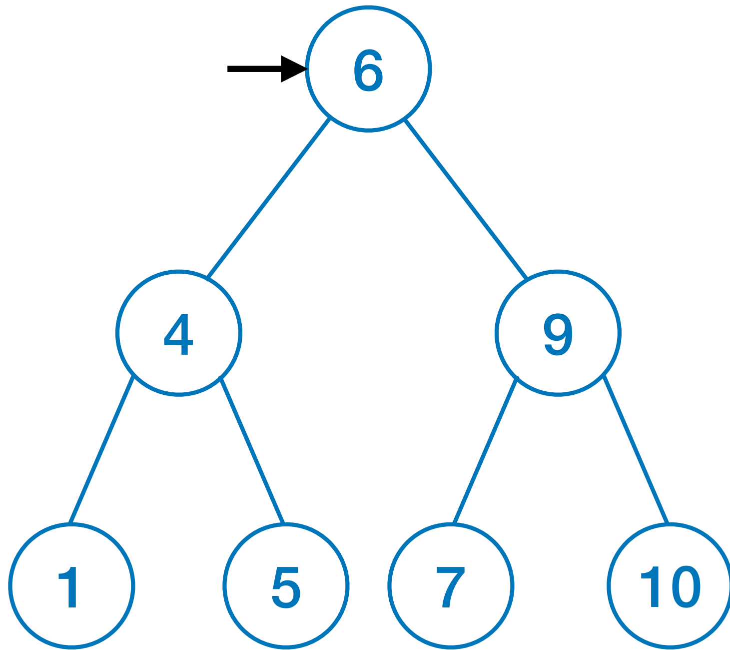
is 5 in this subtree?

Binary Search Trees



is 5 in this subtree?
height of tree is at most *log(n)*

Binary Search Trees




```
1   def bst(node, val):
2       if node == None:
3           return False
4       else:
5           if val == node.label:
6               return True
7           elif val < node.label:
8               return bst(node.left,val)
9           elif val > node.label:
10              return bst(node.right,val)
11
12  bst(root,5)
```