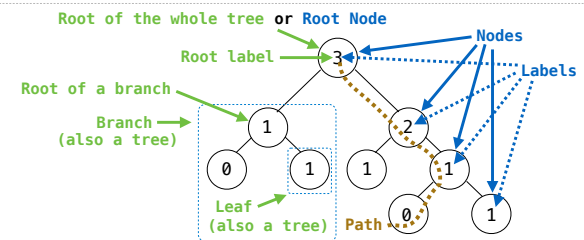


Trees

Announcements

Trees

Tree Abstraction



Recursive description (wooden trees):

- A **tree** has a **root label** and a list of **branches**
- Each **branch** is a **tree**
- A **tree** with zero **branches** is called a **leaf**
- A **tree** starts at the **root**

Relative description (family trees):

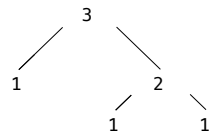
- Each location in a tree is called a **node**
- Each **node** has a **label** that can be any value
- One node can be the **parent/child** of another
- The top node is the **root node**

People often refer to labels by their locations: "each parent is the sum of its children"

Implementing the Tree Abstraction

```
def tree(label, branches=[]):  
    return [label] + branches  
  
def label(tree):  
    return tree[0]  
  
def branches(tree):  
    return tree[1:]
```

- A **tree** has a root **label** and a list of **branches**
- Each branch is a tree



```
>>> tree(3, [tree(1,  
...         tree(2, [tree(1),  
...                 tree(1)])])  
[3, [1], [2, [1], [1]]]
```

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Implementing the Tree Abstraction

```
def tree(label, branches=[]):  
    for branch in branches:  
        assert is_tree(branch)  
    return [label] + list(branches)
```

Verifies the tree definition

```
def label(tree):  
    return tree[0]
```

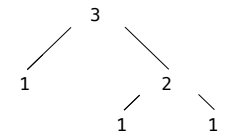
Creates a list from a sequence of branches

```
def branches(tree):  
    return tree[1:]
```

Verifies that tree is bound to a list

```
def is_tree(tree):  
    if type(tree) != list or len(tree) < 1:  
        return False  
    for branch in branches(tree):  
        if not is_tree(branch):  
            return False  
    return True
```

- A **tree** has a root **label** and a list of **branches**
- Each branch is a tree



```
>>> tree(3, [tree(1,  
...         tree(2, [tree(1),  
...                 tree(1)])])  
[3, [1], [2, [1], [1]]]
```

```
def is_leaf(tree):  
    return not branches(tree) (Demo)
```

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Tree Processing

(Demo)

Tree Processing Uses Recursion

Processing a leaf is often the base case of a tree processing function

The recursive case typically makes a recursive call on each branch, then aggregates

```
def count_leaves(t):  
    """Count the leaves of a tree."""  
    if is_leaf(t):  
        return 1  
    else:  
        branch_counts = [count_leaves(b) for b in branches(t)]  
        return sum(branch_counts)
```

(Demo)

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Discussion Question

Implement `leaves`, which returns a list of the leaf labels of a tree

Hint: If you `sum` a list of lists, you get a list containing the elements of those lists

```
>>> sum([ [1], [2, 3], [4] ], [])  def leaves(tree):
[1, 2, 3, 4]                       """Return a list containing the leaf labels of tree.
>>> sum([ [1] ], [])              >>> leaves(fib_tree(5))
[1]                                  [1, 0, 1, 0, 1, 1, 0, 1]
>>> sum([ [[1]], [2] ], [])       >>>>>>
[[1], 2]                             """
                                     if is_leaf(tree):
                                     return [label(tree)]
                                     else:
                                     return sum(List of leaf labels for each branch, [])

branches(tree)                       [b for b in branches(tree)]
leaves(tree)                         [s for s in leaves(tree)]
[branches(b) for b in branches(tree)] [branches(s) for s in leaves(tree)]
[leaves(b) for b in branches(tree)]  [leaves(s) for s in leaves(tree)]
```

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Creating Trees

A function that creates a tree from another tree is typically also recursive

```
def increment_leaves(t):
    """Return a tree like t but with leaf labels incremented."""
    if is_leaf(t):
        return tree(label(t) + 1)
    else:
        bs = [increment_leaves(b) for b in branches(t)]
        return tree(label(t), bs)

def increment(t):
    """Return a tree like t but with all labels incremented."""
    return tree(label(t) + 1, [increment(b) for b in branches(t)])
```

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Example: Printing Trees

(Demo)

Example: Summing Paths

(Demo)

Example: Counting Paths

Count Paths that have a Total Label Sum

```
def count_paths(t, total):
    """Return the number of paths from the root to any node in tree t
    for which the labels along the path sum to total.

    >>> t = tree(3, [tree(-1), tree(1, [tree(2, [tree(1)]), tree(3)]), tree(1, [tree(-1)])])
    >>> count_paths(t, 3) ◀
    2
    >>> count_paths(t, 4) ◀
    2
    >>> count_paths(t, 5)
    0
    >>> count_paths(t, 6)
    1
    >>> count_paths(t, 7) ◀
    2
    """
    if label(t) == total:
        found = 1
    else:
        found = 0
    return found + sum([count_paths(b, total - label(t)) for b in branches(t)])
```

