Data Abstraction

Announcements

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All Programmers

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All Programmers

Great Programmers

Rational Numbers

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$$\frac{\texttt{numerator}}{\texttt{denominator}}$$

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Exact representation of fractions

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A pair of integers

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- rational(n, d) returns a rational number x

Rational Numbers

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Exact representation of fractions

A pair of integers

As soon as division occurs, the exact representation may be lost! (Demo)

Assume we can compose and decompose rational numbers:

- rational(n, d) returns a rational number x
- numer(x) returns the numerator of x

Rational Numbers

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Exact representation of fractions

A pair of integers

As soon as division occurs, the exact representation may be lost! (Demo)

Assume we can compose and decompose rational numbers:

- rational(n, d) returns a rational number x
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Rational Numbers

numerator
———————
denominator

Exact representation of fractions

A pair of integers

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Assume we can compose and decompose rational numbers:

Constructor

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Rational Numbers

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Exact representation of fractions

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As soon as division occurs, the exact representation may be lost! (Demo)

Assume we can compose and decompose rational numbers:

Constructor
- rational(n, d) returns a rational number x

Selectors
- numer(x) returns the numerator of x
- denom(x) returns the denominator of x

Rational Number Arithmetic

Example

General Form

Rational Number Arithmetic

$$\frac{3}{2} * \frac{3}{5}$$

Example

General Form

Rational Number Arithmetic

$$\frac{3}{2} * \frac{3}{5} = \frac{9}{10}$$

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$$\frac{nx}{dx} * \frac{ny}{dy}$$

General Form

Rational Number Arithmetic

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Example

$$\frac{nx}{dx} * \frac{ny}{dy} = \frac{nx*ny}{dx*dy}$$

General Form

Rational Number Arithmetic

$$\frac{3}{2} * \frac{3}{5} = \frac{9}{10}$$

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$$\frac{nx}{dx} * \frac{ny}{dy} = \frac{nx*ny}{dx*dy}$$

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General Form

Rational Number Arithmetic Implementation

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- `rational(n, d)` returns a rational number `x`
- `numer(x)` returns the numerator of `x`
- `denom(x)` returns the denominator of `x`

Rational Number Arithmetic Implementation

```
def mul_rational(x, y):
    return rational(numer(x) * numer(y),
                    denom(x) * denom(y))
```

$$\frac{nx}{dx} * \frac{ny}{dy} = \frac{nx*ny}{dx*dy}$$

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These functions implement an abstract representation for rational numbers

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def mul_rational(x, y):
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```

Constructor

Selectors

$$\frac{nx}{dx} * \frac{ny}{dy} = \frac{nx*ny}{dx*dy}$$

```
def add_rational(x, y):
    nx, dx = numer(x), denom(x)
    ny, dy = numer(y), denom(y)
    return rational(nx * dy + ny * dx, dx * dy)
```

$$\frac{nx}{dx} + \frac{ny}{dy} = \frac{nx*dy + ny*dx}{dx*dy}$$

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def mul_rational(x, y):
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$$\frac{nx}{dx} * \frac{ny}{dy} = \frac{nx*ny}{dx*dy}$$

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def add_rational(x, y):
    nx, dx = numer(x), denom(x)
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    return rational(nx * dy + ny * dx, dx * dy)

def print_rational(x):
    print(numer(x), '/', denom(x))
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def add_rational(x, y):
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    return rational(nx * dy + ny * dx, dx * dy)

def print_rational(x):
    print(numer(x), '/', denom(x))

def rationals_are_equal(x, y):
    return numer(x) * denom(y) == numer(y) * denom(x)
```

$$\frac{nx}{dx} + \frac{ny}{dy} = \frac{nx*dy + ny*dx}{dx*dy}$$

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Representing Rational Numbers

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```
def rational(n, d):
    """Construct a rational number that represents N/D."""
    return [n, d]
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Construct a list

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Construct a list

```
def numer(x):
    """Return the numerator of rational number X."""
    return x[0]
```

Representing Rational Numbers

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def rational(n, d):
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Construct a list

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def numer(x):
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    """Return the denominator of rational number X."""
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Select item from a list

Representing Rational Numbers

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Select item from a list

(Demo)

Reducing to Lowest Terms

Example:

Reducing to Lowest Terms

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$$\frac{3}{2} * \frac{5}{3}$$

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$$\frac{15}{6} * \frac{1/3}{1/3} = \frac{5}{2}$$

Reducing to Lowest Terms

Example:

$$\frac{3}{2} * \frac{5}{3} = \boxed{\frac{5}{2}} \qquad \frac{2}{5} + \frac{1}{10}$$

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```
from math import gcd
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Greatest common divisor

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Abstraction Barriers

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Parts of the program that...	Treat rationals as...	Using...

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Use rational numbers to perform computation		

Abstraction Barriers

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Abstraction Barriers

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Implementation of lists

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Implementation of lists

Violating Abstraction Barriers

```
add_rational( [1, 2], [1, 4] )

def divide_rational(x, y):
    return [ x[0] * y[1], x[1] * y[0] ]
```

Violating Abstraction Barriers

Does not use
constructors

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Twice!

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Violating Abstraction Barriers

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No selectors!

And no constructor!

Violating Abstraction Barriers

Data Representations

What are Data?

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- We need to guarantee that constructor and selector functions work together to specify the right behavior

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You can recognize an abstract data representation by its behavior

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You can recognize an abstract data representation by its behavior

(Demo)

Rationals Implemented as Functions

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```
def rational(n, d):
    def select(name):
        if name == 'n':
            return n
        elif name == 'd':
            return d
    return select
```

```
def numer(x):
    return x('n')

def denom(x):
    return x('d')
```

Rationals Implemented as Functions

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This function represents a rational number

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Rationals Implemented as Functions

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def rational(n, d):
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            return n
        elif name == 'd':
            return d
    return select
```

This function represents a rational number

Constructor is a higher-order function

```
def numer(x):
    return x('n')

def denom(x):
    return x('d')
```

Rationals Implemented as Functions

```
def rational(n, d):
    def select(name):
        if name == 'n':
            return n
        elif name == 'd':
            return d
    return select
```

This function represents a rational number

Constructor is a higher-order function

```
def numer(x):
    return x('n')

def denom(x):
    return x('d')
```

Selector calls x

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Selector calls x

```
x = rational(3, 8)
numer(x)
```

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Constructor is a higher-order function

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def numer(x):
    return x('n')

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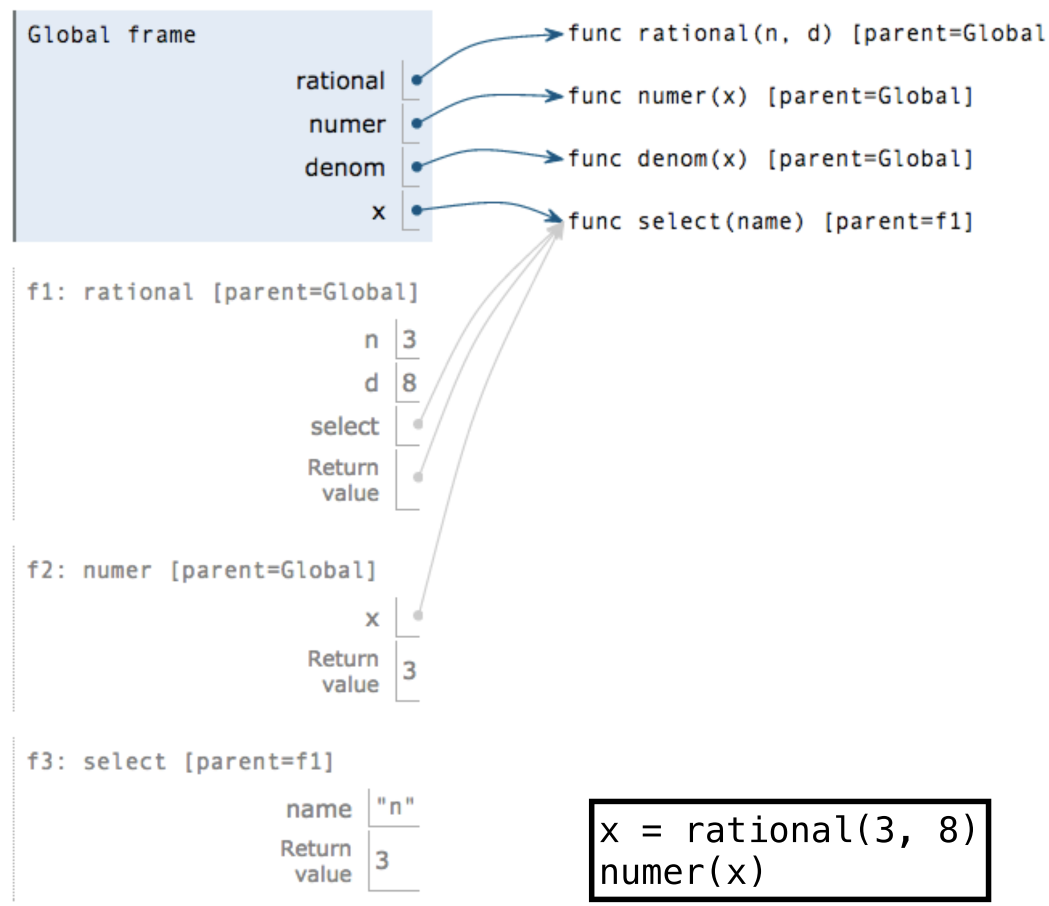


```
x = rational(3, 8)
numer(x)
```

pythontutor.com/composingprograms.html#code=def%20rational%28n,
%20d%29%3A%0A%20%20%20%20def%20select%28name%29%3A%0A%20%20%20%20%20%20%20%20if%20name%20%3D%3D%20'n'%3A%0A%20%20%20%20%20%20%20%20%20%20%20%20return%20n%0A%20%20%20%20%20%20%20%20elif%20name%20%3D%3D%20'd'%3A%0A%20%20%20%20%20%20%20%20%20%20%20%20return%20d%0A%20%20%20%20return%20select%
0A%20%20%20%20%0Adef%20numer%28x%29%3A%0A%20%20%20%20return%20x%28'n'%29%0A%0Adef%20denom%28x%29%3A%0A%20%20%20%20return%20x%28'd'%29%0A%20%20%20%20%0Ax%20%3D%20rational%283,%208%29%0Anumer%28x%29&mode=display&origin=composingprograms.js&cumulative=true&py=3&rawInputLstJSON=[]&curInstr=0