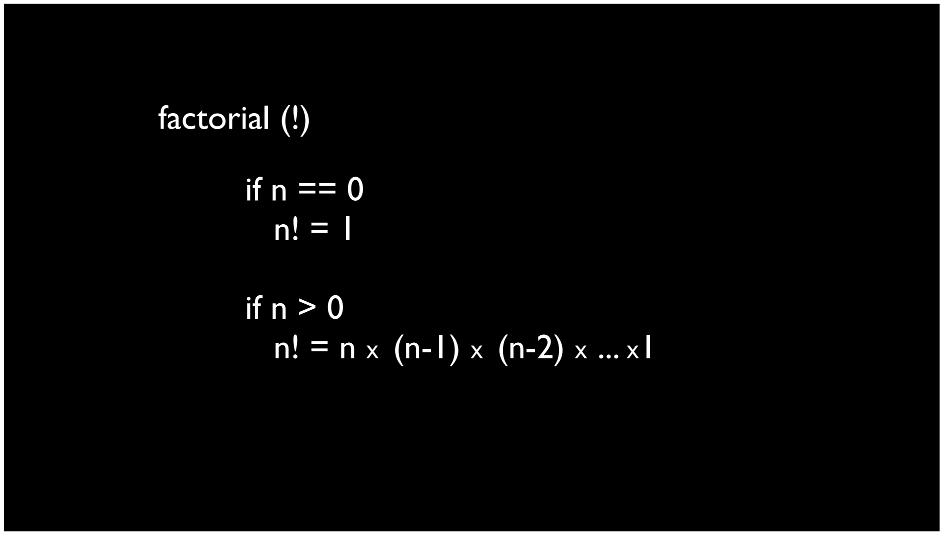
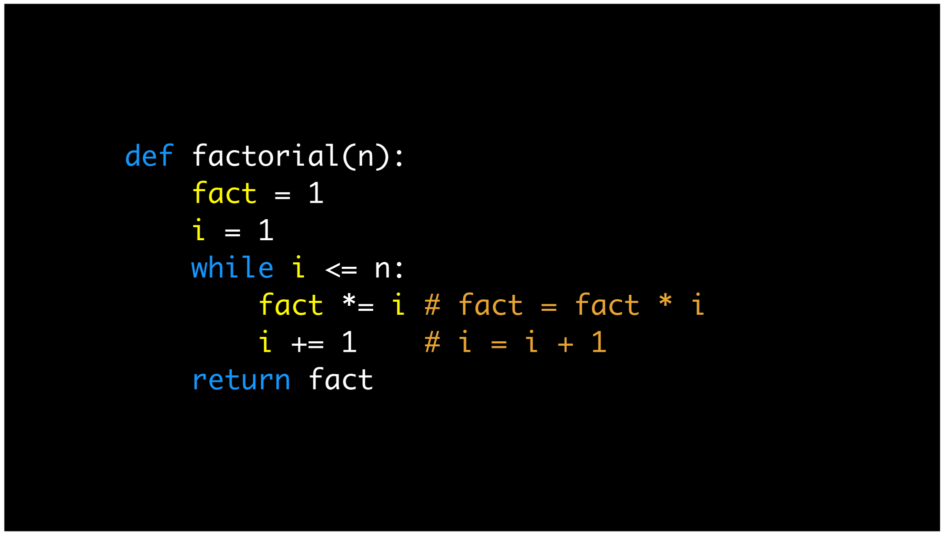
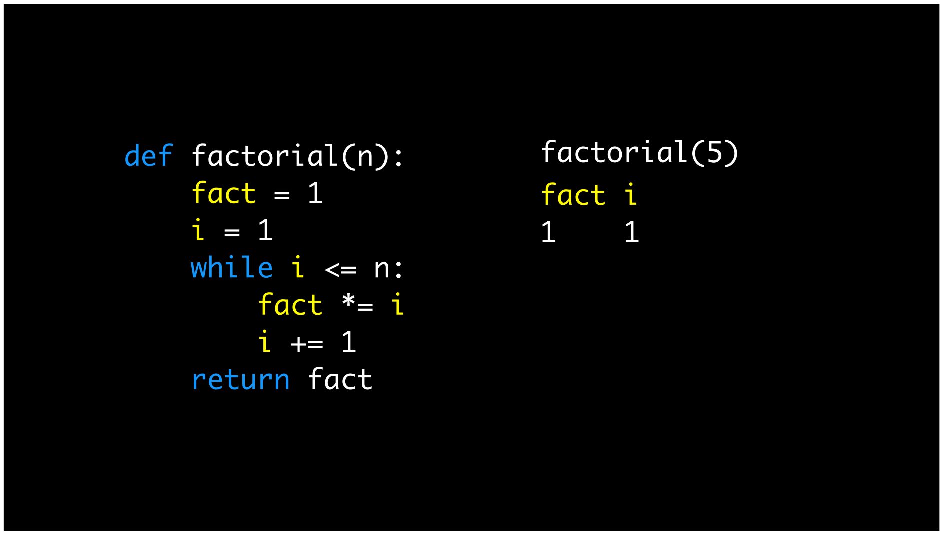


factorial (!)

```
if n == 0
  n! = 1

if n > 0
  n! = n x (n-1) x (n-2) x ... x 1
```



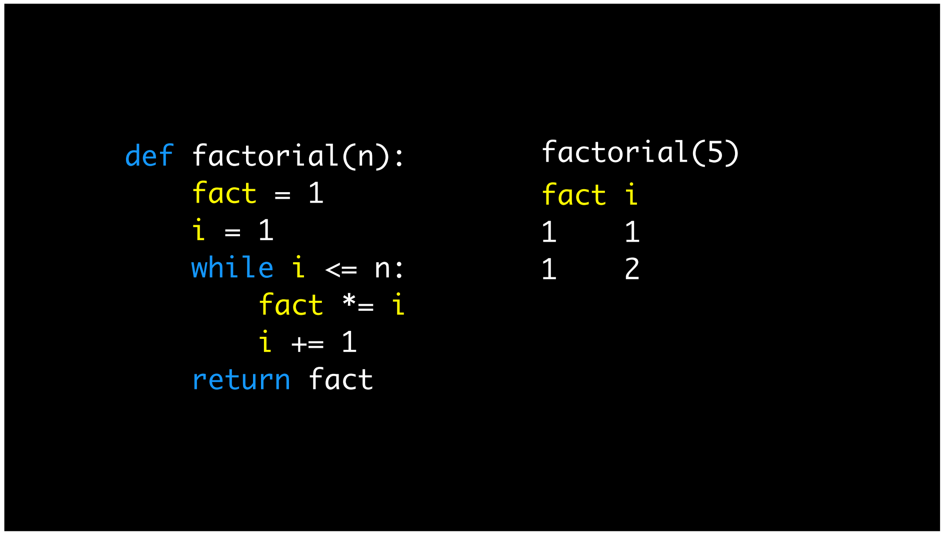
```python
def factorial(n):
    fact = 1
    i = 1
    while i <= n:
        fact *= i # fact = fact * i
        i += 1    # i = i + 1
    return fact
```



```python
def factorial(n):
    fact = 1
    i = 1
    while i <= n:
        fact *= i
        i += 1
    return fact
```

factorial(5)

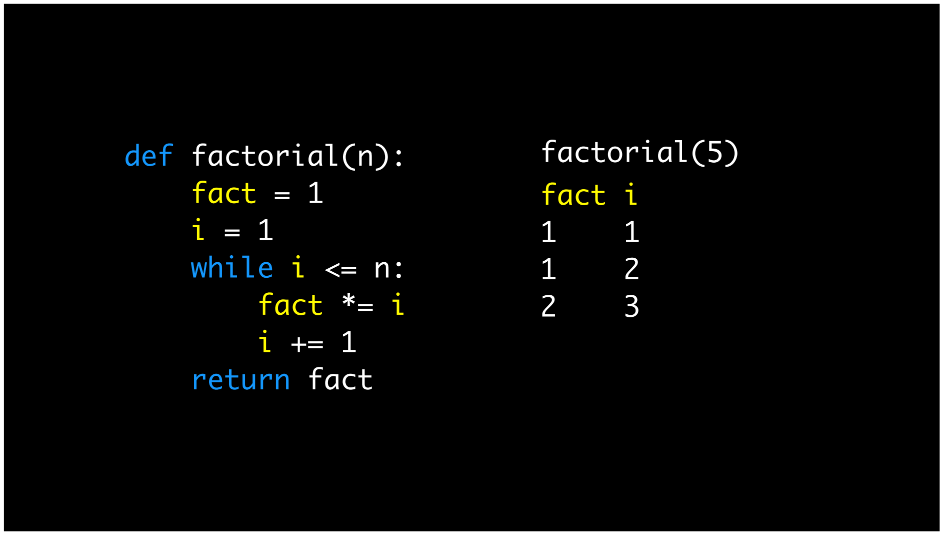
fact	i
1	1



```python
def factorial(n):
    fact = 1
    i = 1
    while i <= n:
        fact *= i
        i += 1
    return fact
```

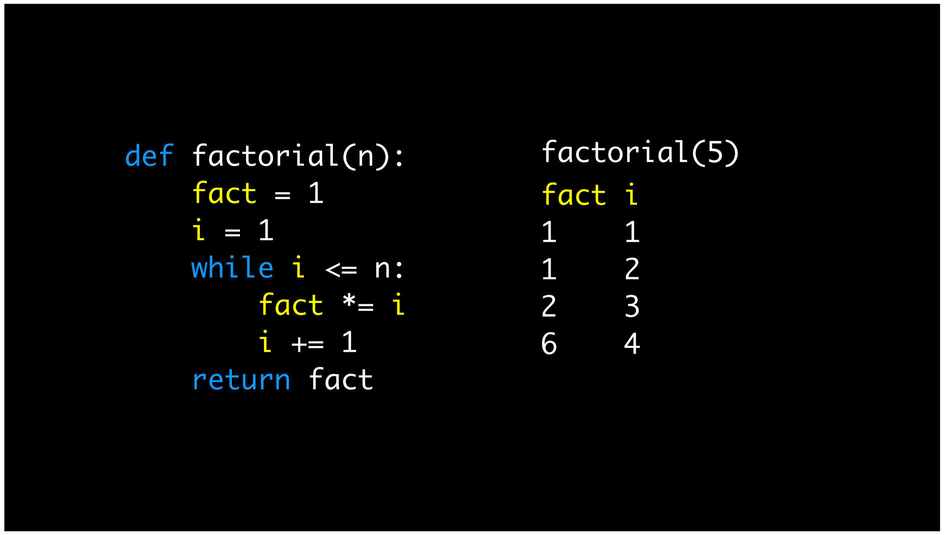
factorial(5)

fact	i
1	1
1	2



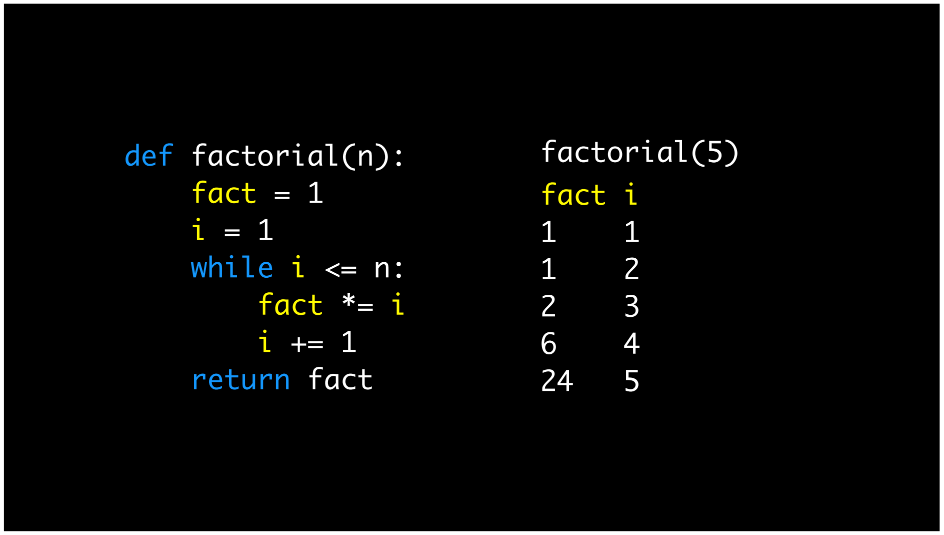
```
def factorial(n):
    fact = 1
    i = 1
    while i <= n:
        fact *= i
        i += 1
    return fact
```

```
factorial(5)
fact i
1    1
1    2
2    3
```



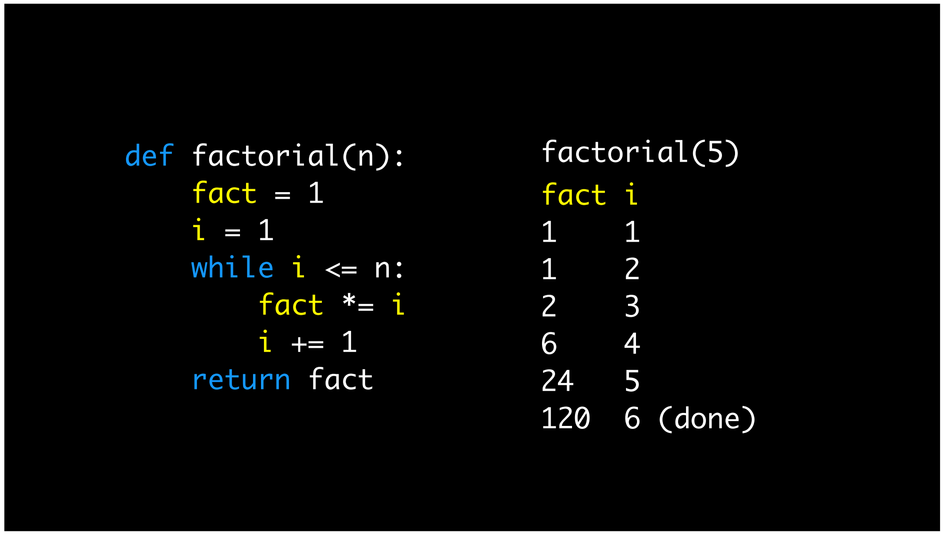
```
def factorial(n):
    fact = 1
    i = 1
    while i <= n:
        fact *= i
        i += 1
    return fact
```

```
factorial(5)
fact i
1    1
1    2
2    3
6    4
```



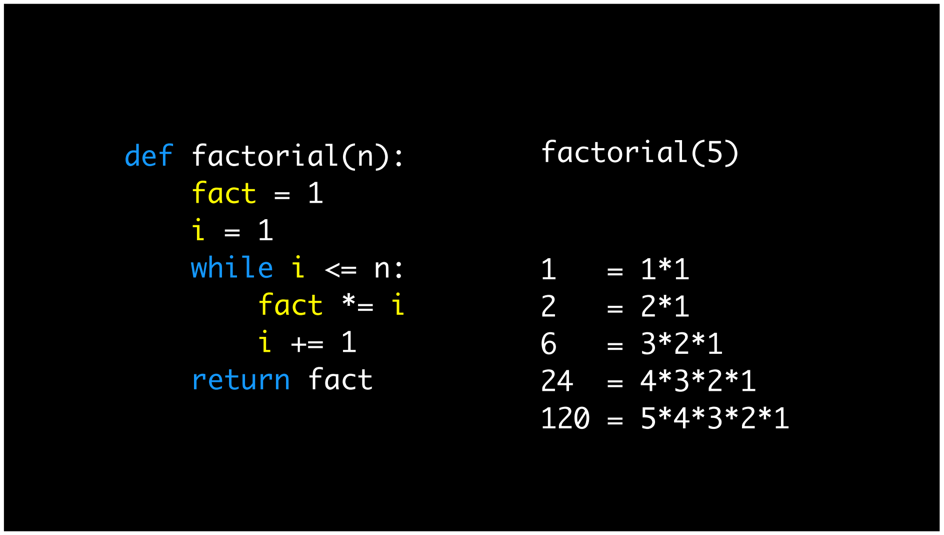
```
def factorial(n):
    fact = 1
    i = 1
    while i <= n:
        fact *= i
        i += 1
    return fact
```

```
factorial(5)
fact i
1    1
1    2
2    3
6    4
24   5
```



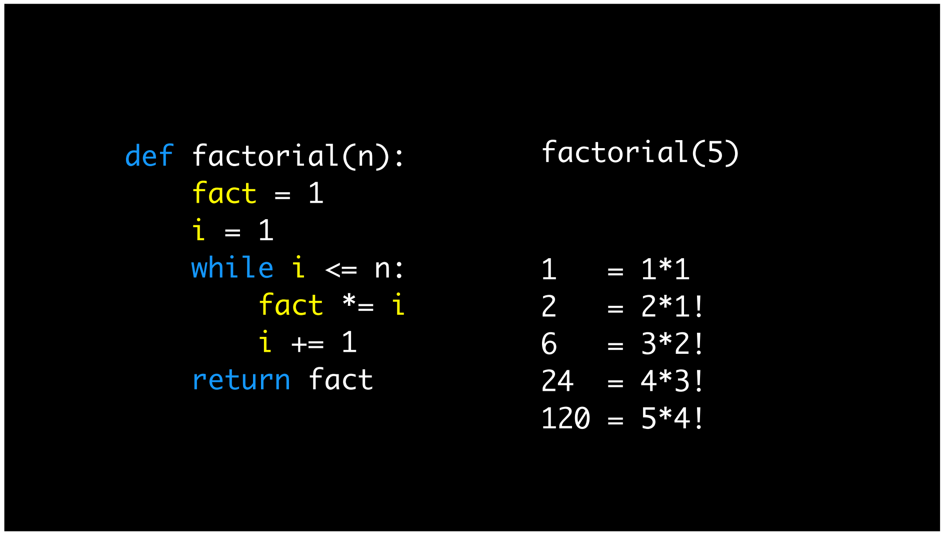
```
def factorial(n):
    fact = 1
    i = 1
    while i <= n:
        fact *= i
        i += 1
    return fact
```

```
factorial(5)
fact i
1    1
1    2
2    3
6    4
24   5
120  6 (done)
```



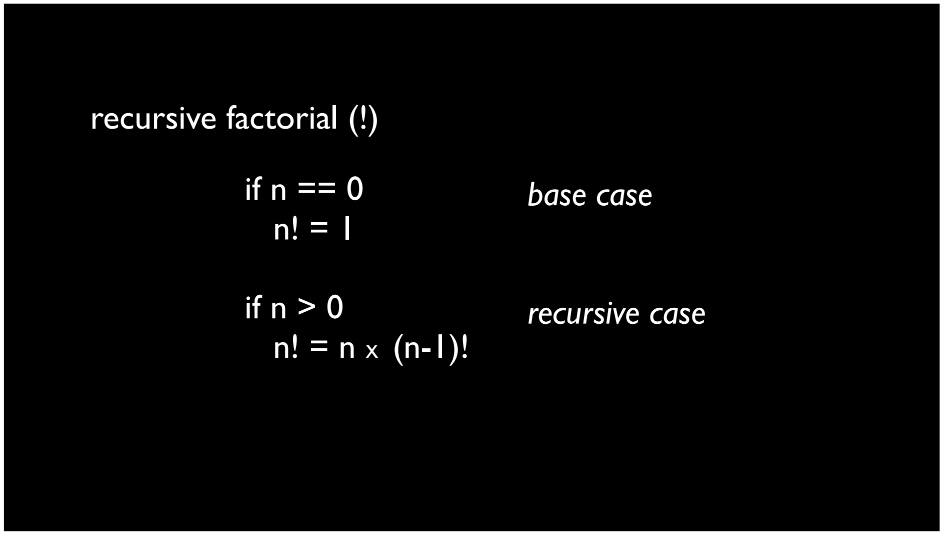
```
def factorial(n):
    fact = 1
    i = 1
    while i <= n:
        fact *= i
        i += 1
    return fact
```

```
factorial(5)


1   = 1*1
2   = 2*1
6   = 3*2*1
24  = 4*3*2*1
120 = 5*4*3*2*1
```



```
def factorial(n):
    fact = 1
    i = 1
    while i <= n:
        fact *= i
        i += 1
    return fact
```

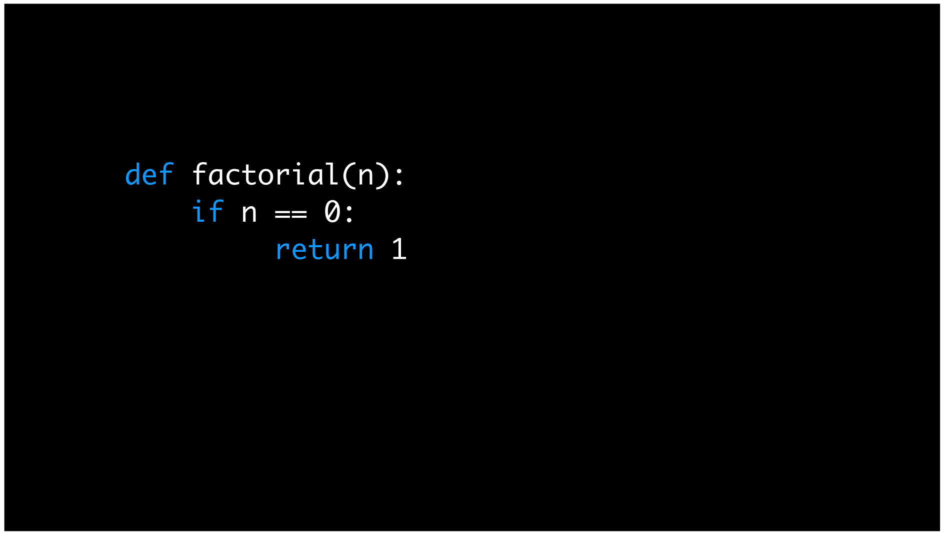
```
factorial(5)


1   = 1*1
2   = 2*1!
6   = 3*2!
24  = 4*3!
120 = 5*4!
```



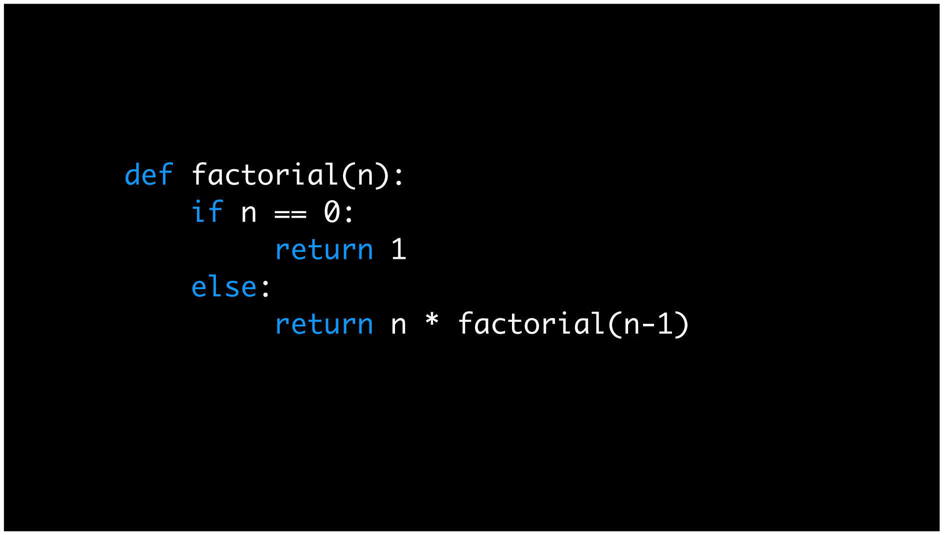
recursive factorial (!)

if n == 0
  n! = 1 — *base case*

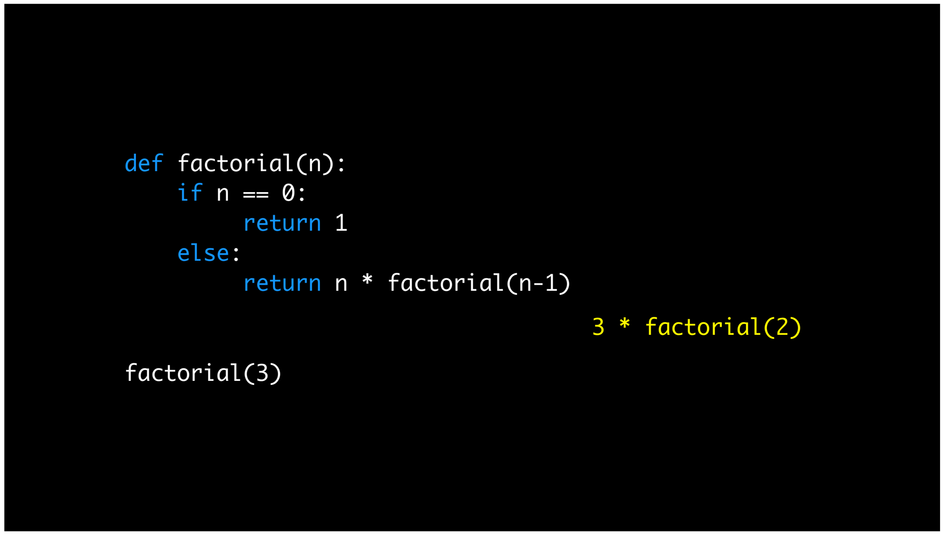
if n > 0
  n! = n x (n-1)! — *recursive case*



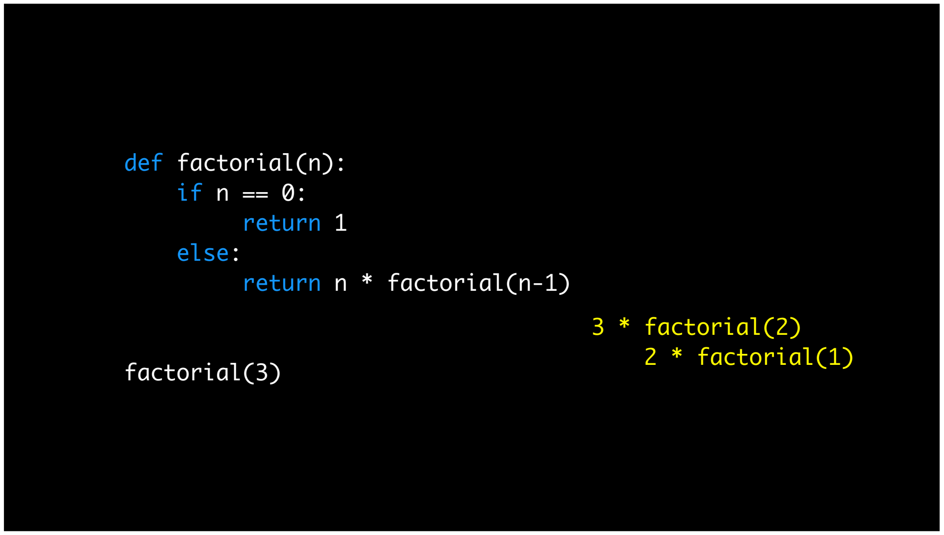
```
def factorial(n):
    if n == 0:
        return 1
```



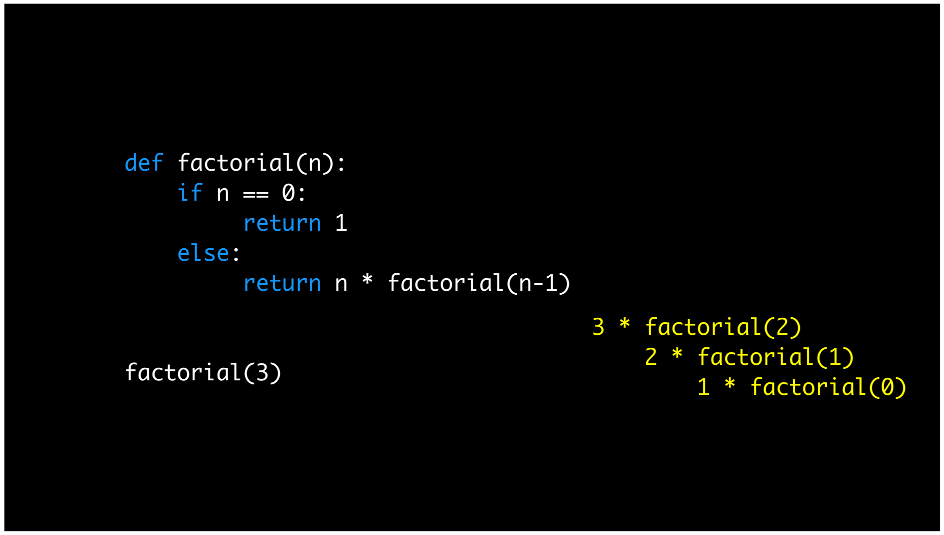
```
def factorial(n):
    if n == 0:
        return 1
    else:
        return n * factorial(n-1)
```



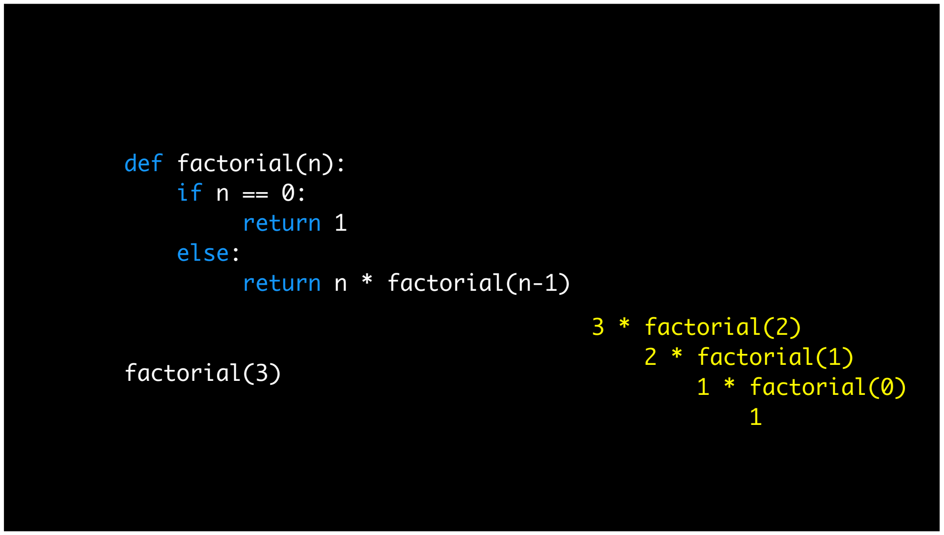
```
def factorial(n):
    if n == 0:
        return 1
    else:
        return n * factorial(n-1)

                                   3 * factorial(2)

factorial(3)
```



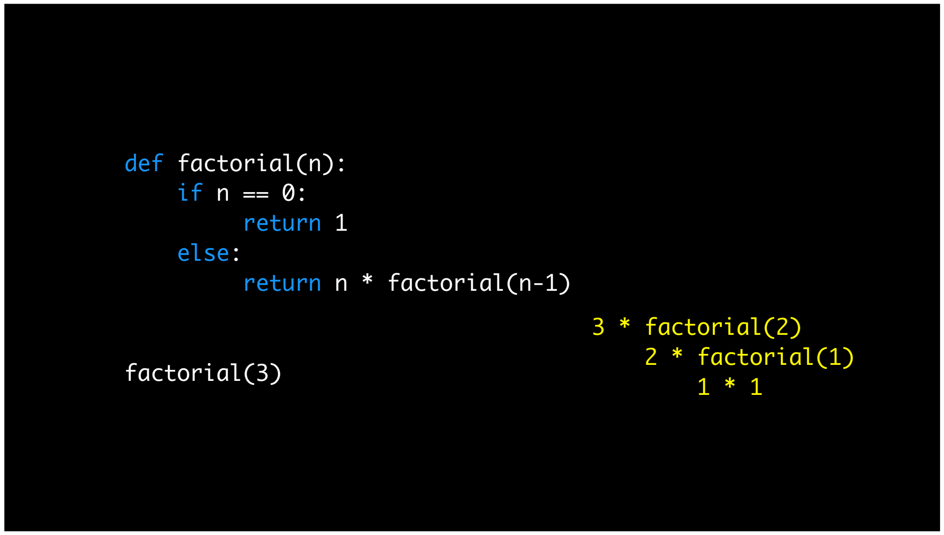
```
def factorial(n):
    if n == 0:
        return 1
    else:
        return n * factorial(n-1)

                                   3 * factorial(2)
                                       2 * factorial(1)
factorial(3)
```



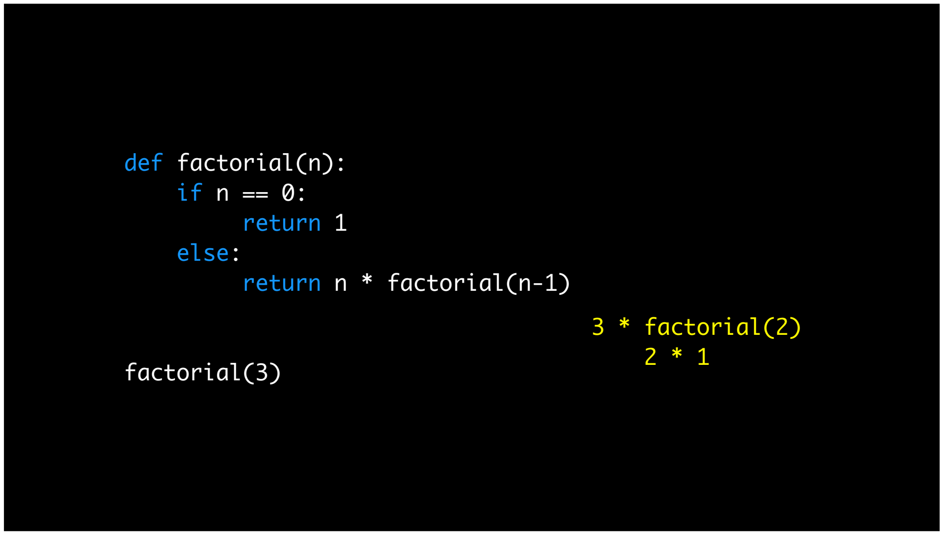
```
def factorial(n):
    if n == 0:
        return 1
    else:
        return n * factorial(n-1)

                                   3 * factorial(2)
                                       2 * factorial(1)
factorial(3)
                                           1 * factorial(0)
```



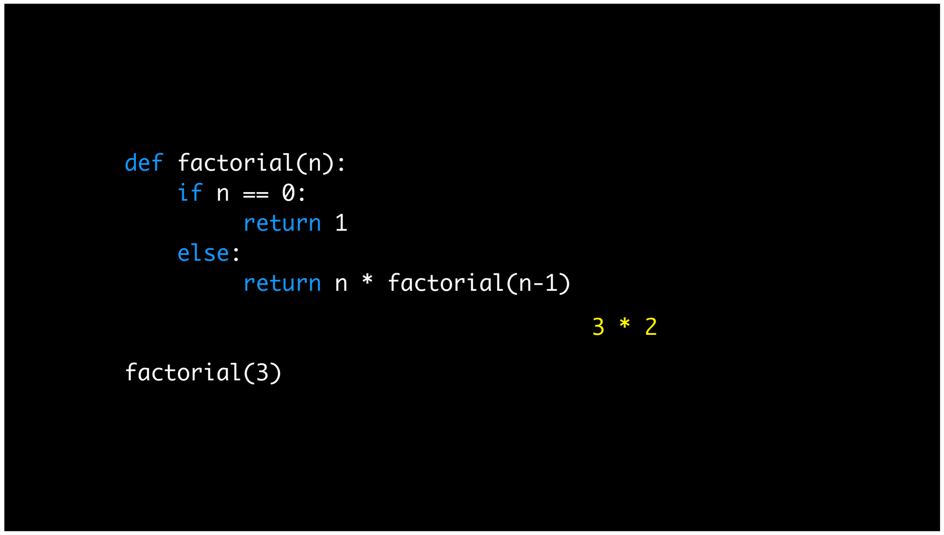
```
def factorial(n):
    if n == 0:
        return 1
    else:
        return n * factorial(n-1)

                                    3 * factorial(2)
factorial(3)                            2 * factorial(1)
                                            1 * factorial(0)
                                                1
```



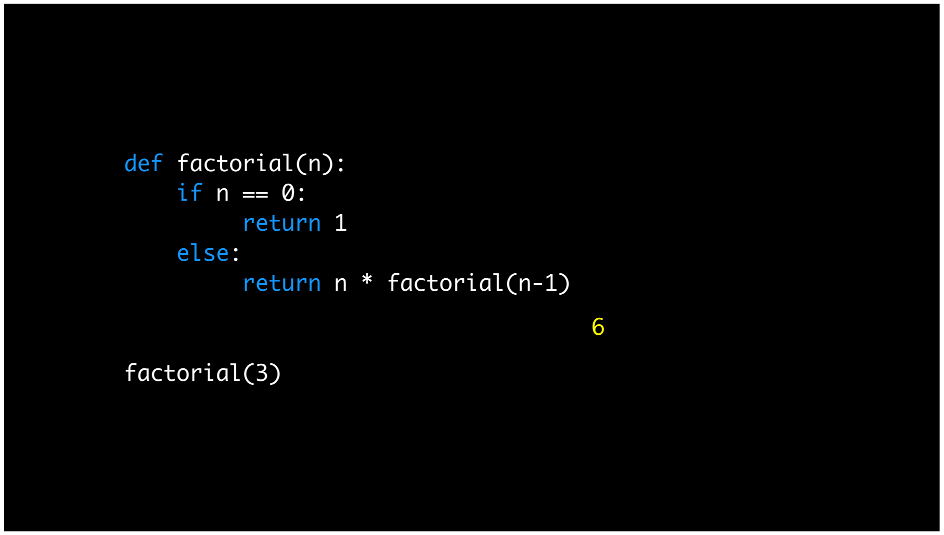
```
def factorial(n):
    if n == 0:
        return 1
    else:
        return n * factorial(n-1)

                                    3 * factorial(2)
factorial(3)                            2 * factorial(1)
                                            1 * 1
```



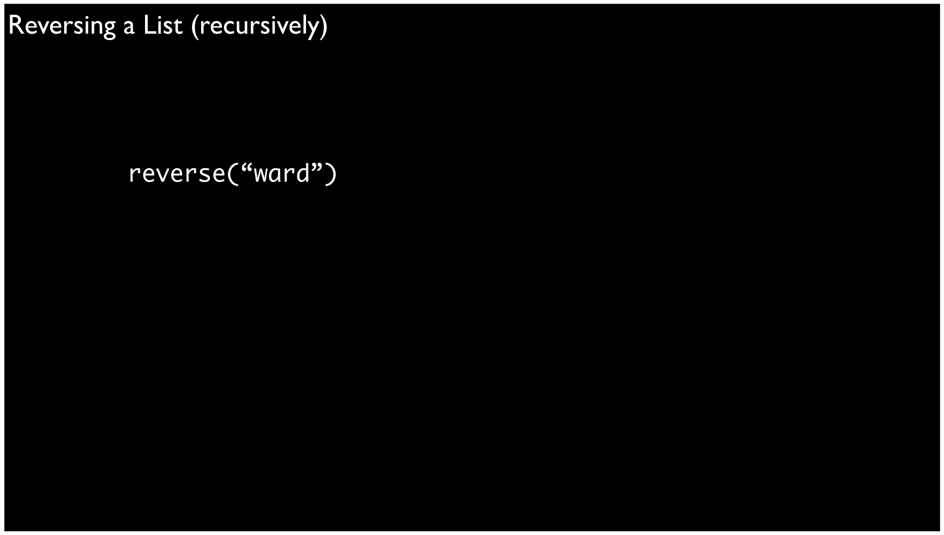
```
def factorial(n):
    if n == 0:
        return 1
    else:
        return n * factorial(n-1)

                                    3 * factorial(2)
factorial(3)                            2 * 1
```



```
def factorial(n):
    if n == 0:
        return 1
    else:
        return n * factorial(n-1)

                                    3 * 2

factorial(3)
```

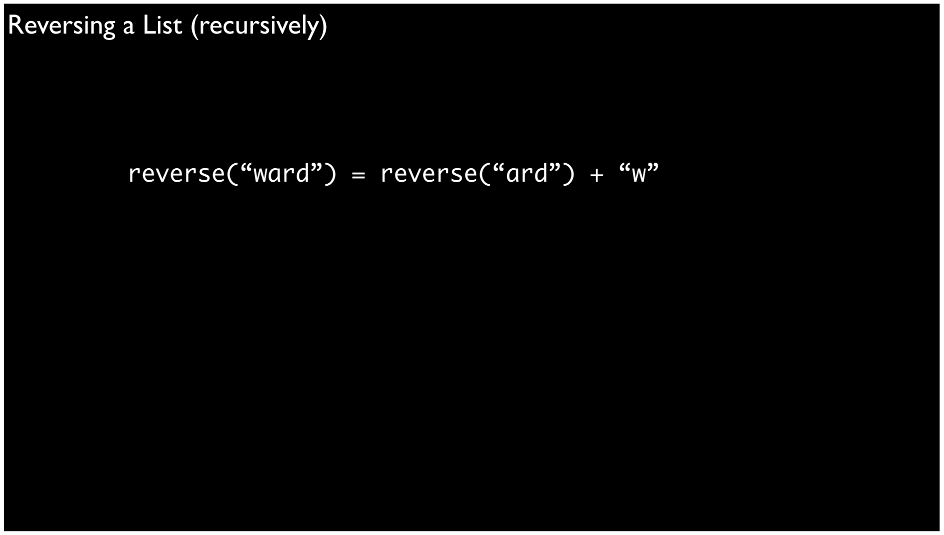


```
def factorial(n):
    if n == 0:
        return 1
    else:
        return n * factorial(n-1)
                                      6

factorial(3)
```



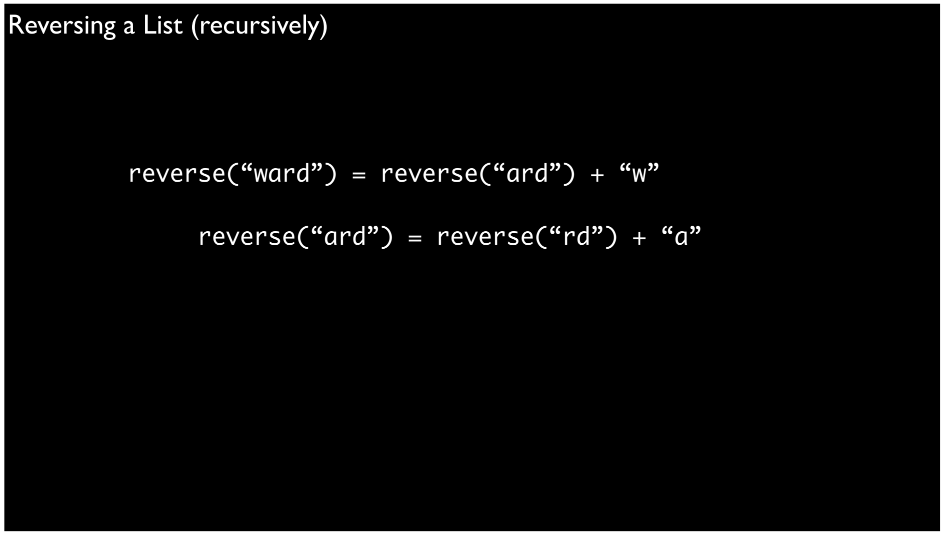
Reversing a List (recursively)

```
reverse(“ward”)
```



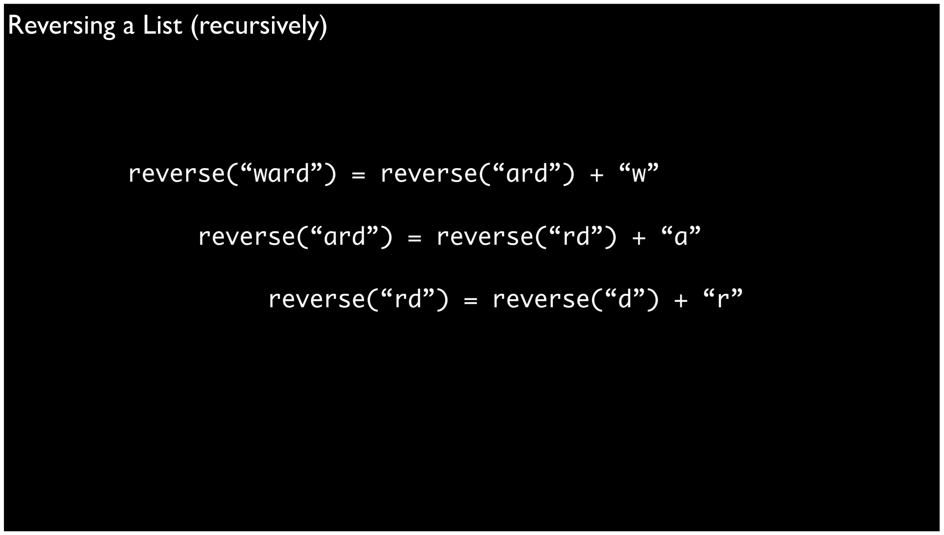
Reversing a List (recursively)

```
reverse(“ward”) = reverse(“ard”) + “w”
```



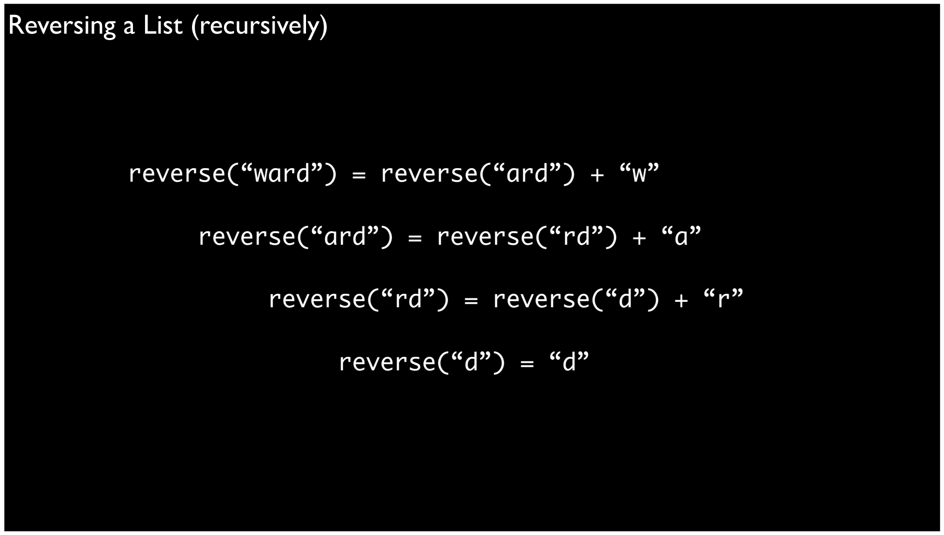
Reversing a List (recursively)

```
reverse(“ward”) = reverse(“ard”) + “w”

     reverse(“ard”) = reverse(“rd”) + “a”
```



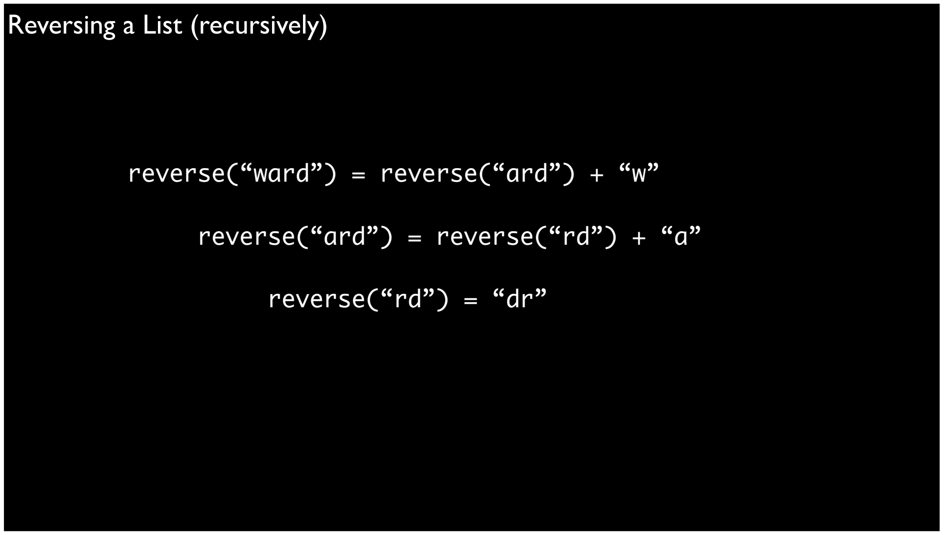
Reversing a List (recursively)

```
reverse(“ward”) = reverse(“ard”) + “w”

     reverse(“ard”) = reverse(“rd”) + “a”

          reverse(“rd”) = reverse(“d”) + “r”
```



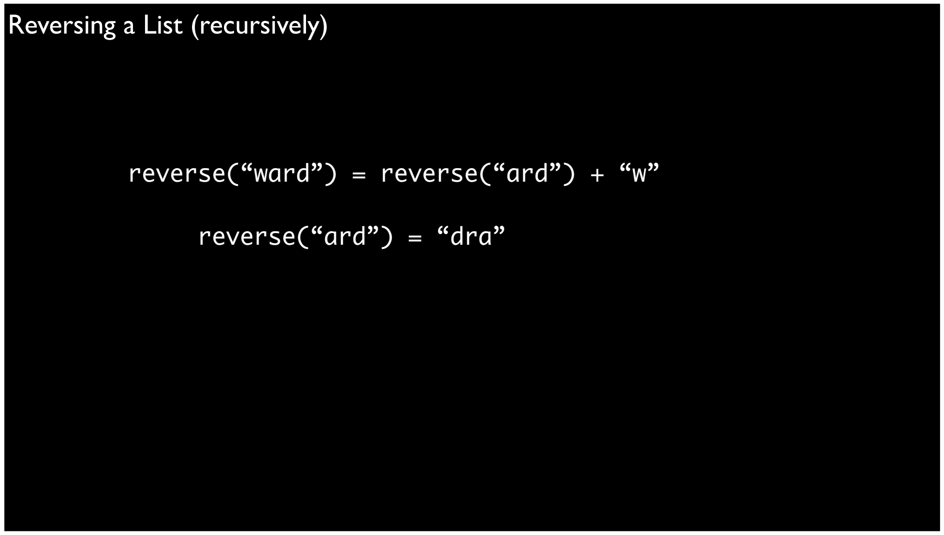
Reversing a List (recursively)

```
reverse(“ward”) = reverse(“ard”) + “w”

     reverse(“ard”) = reverse(“rd”) + “a”

          reverse(“rd”) = reverse(“d”) + “r”

               reverse(“d”) = “d”
```



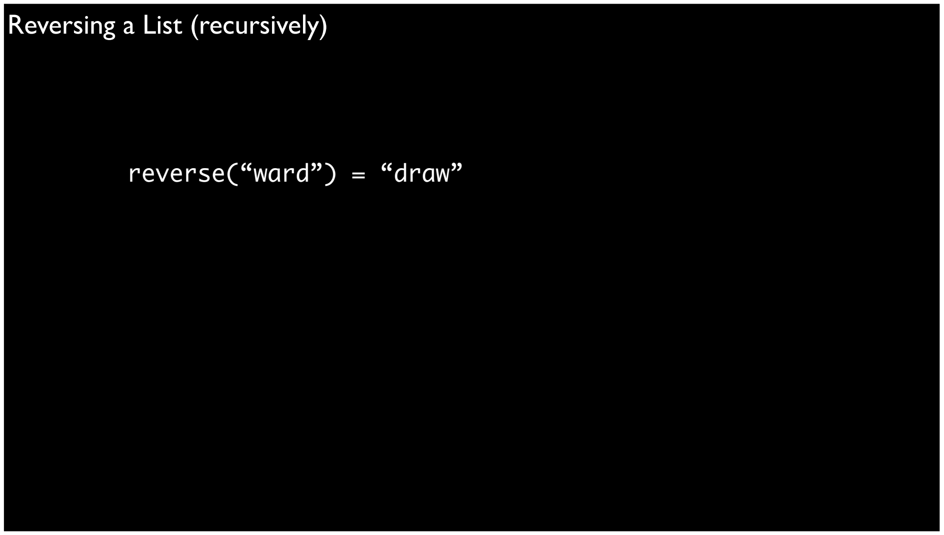
Reversing a List (recursively)

```
reverse(“ward”) = reverse(“ard”) + “w”

     reverse(“ard”) = reverse(“rd”) + “a”

          reverse(“rd”) = “dr”
```



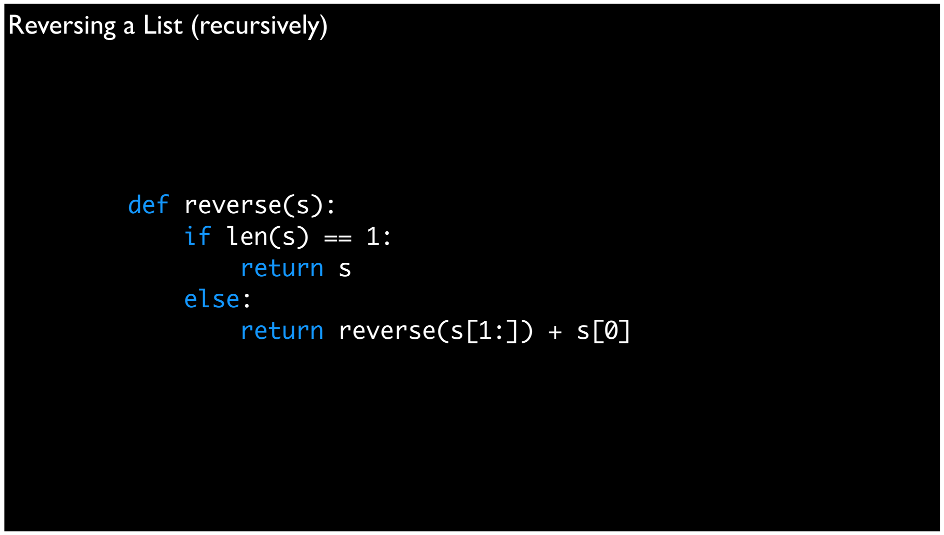
Reversing a List (recursively)

```
reverse(“ward”) = reverse(“ard”) + “w”

     reverse(“ard”) = “dra”
```



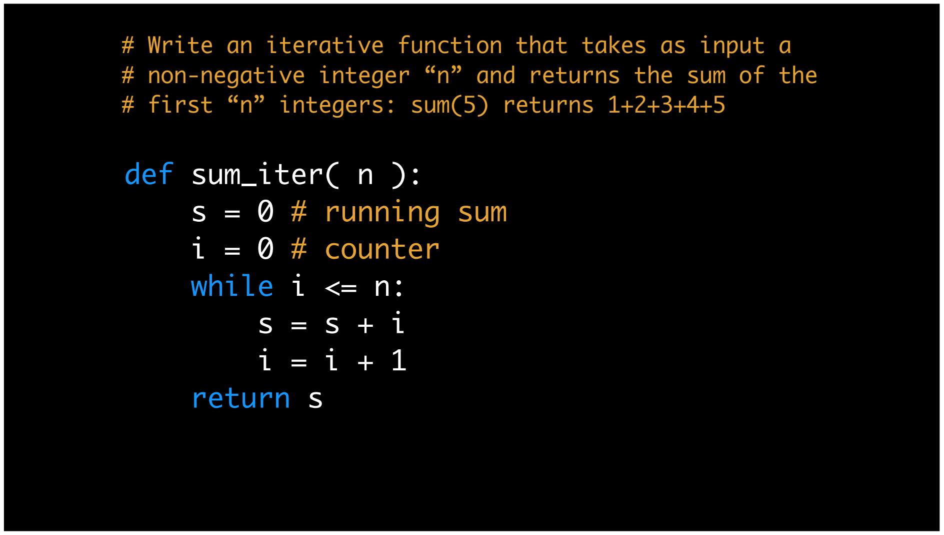
Reversing a List (recursively)

```
reverse(“ward”) = “draw”
```

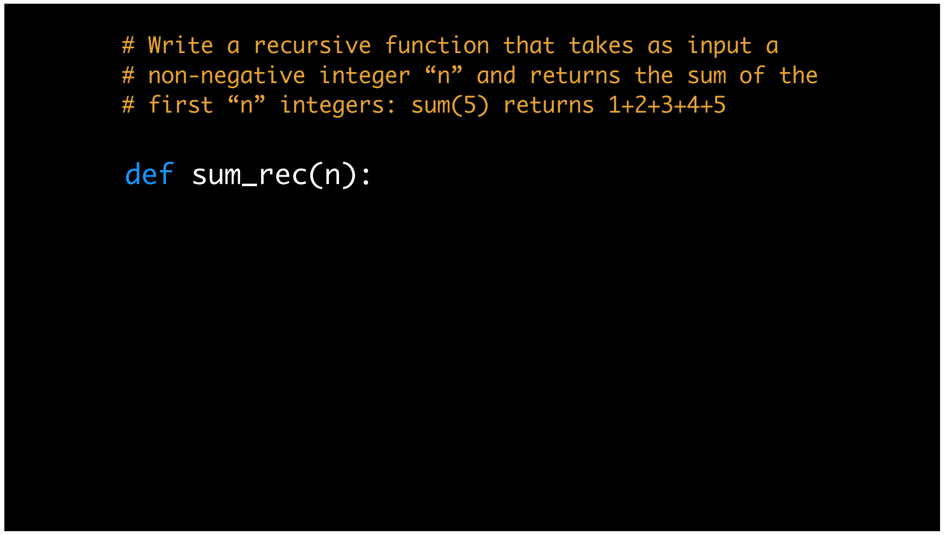


Reversing a List (recursively)

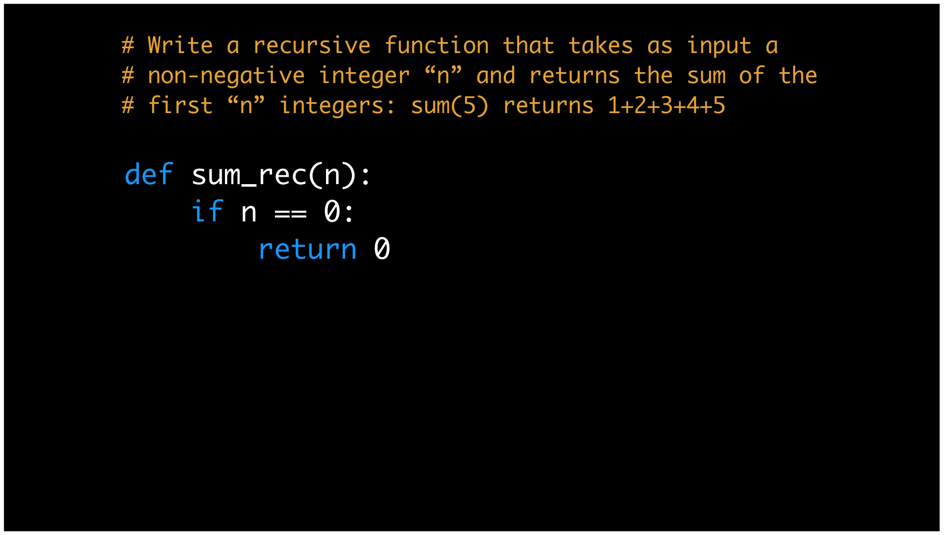
```
def reverse(s):
    if len(s) == 1:
        return s
    else:
        return reverse(s[1:]) + s[0]
```



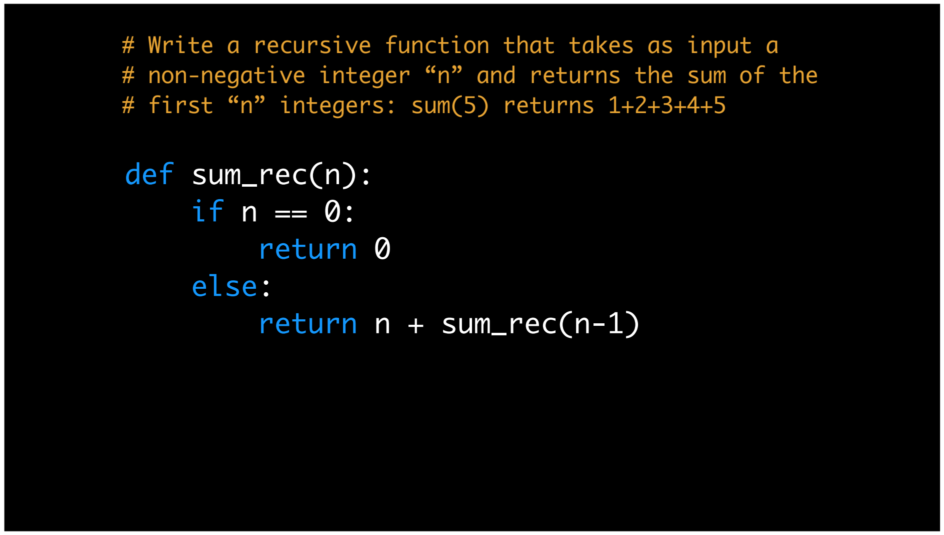
```
# Write an iterative function that takes as input a
# non-negative integer “n” and returns the sum of the
# first “n” integers: sum(5) returns 1+2+3+4+5

def sum_iter( n ):
    s = 0 # running sum
    i = 0 # counter
    while i <= n:
        s = s + i
        i = i + 1
    return s
```



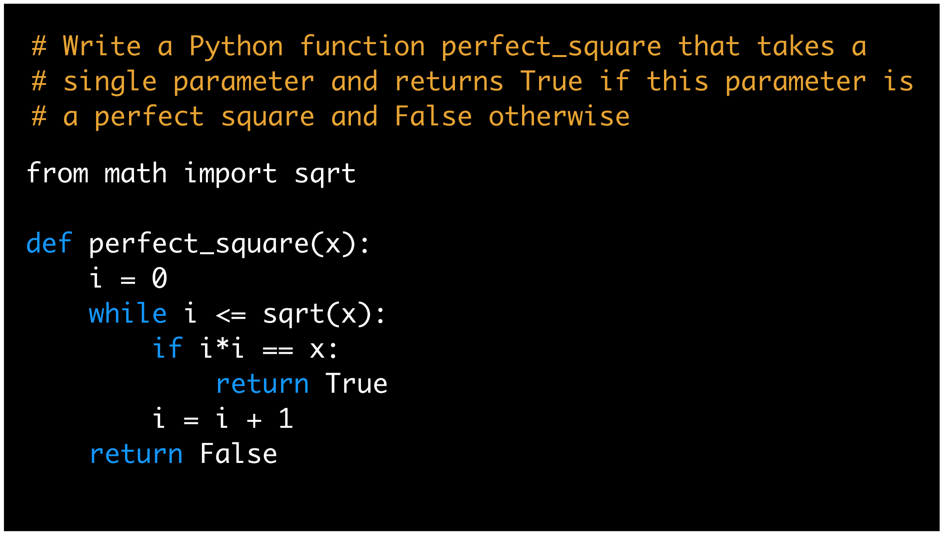
```
# Write a recursive function that takes as input a
# non-negative integer “n” and returns the sum of the
# first “n” integers: sum(5) returns 1+2+3+4+5

def sum_rec(n):
```



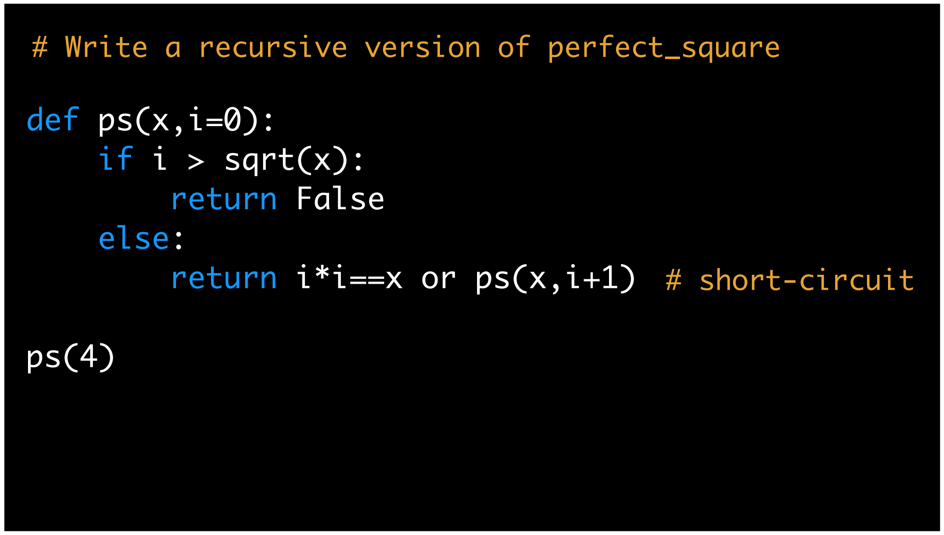
```
# Write a recursive function that takes as input a
# non-negative integer “n” and returns the sum of the
# first “n” integers: sum(5) returns 1+2+3+4+5

def sum_rec(n):
    if n == 0:
        return 0
```



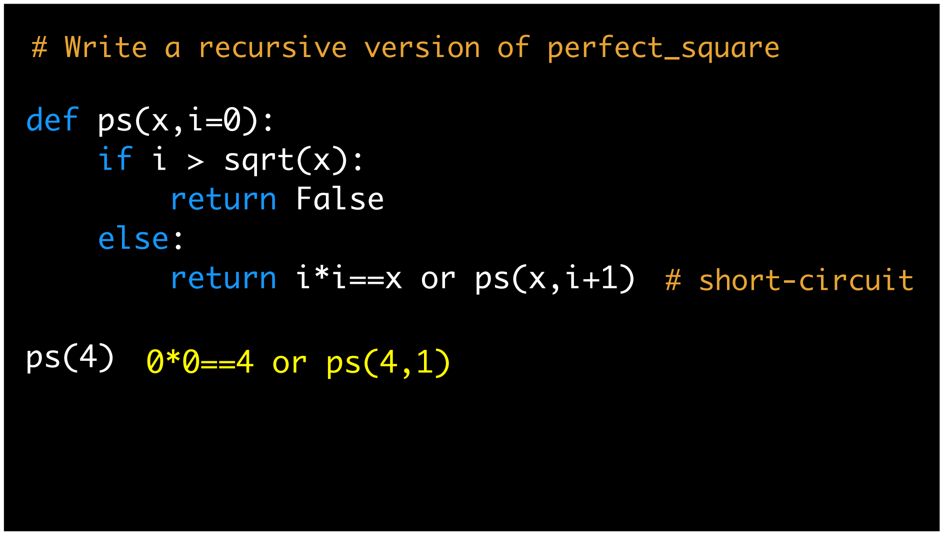
```
# Write a recursive function that takes as input a
# non-negative integer “n” and returns the sum of the
# first “n” integers: sum(5) returns 1+2+3+4+5

def sum_rec(n):
    if n == 0:
        return 0
    else:
        return n + sum_rec(n-1)
```



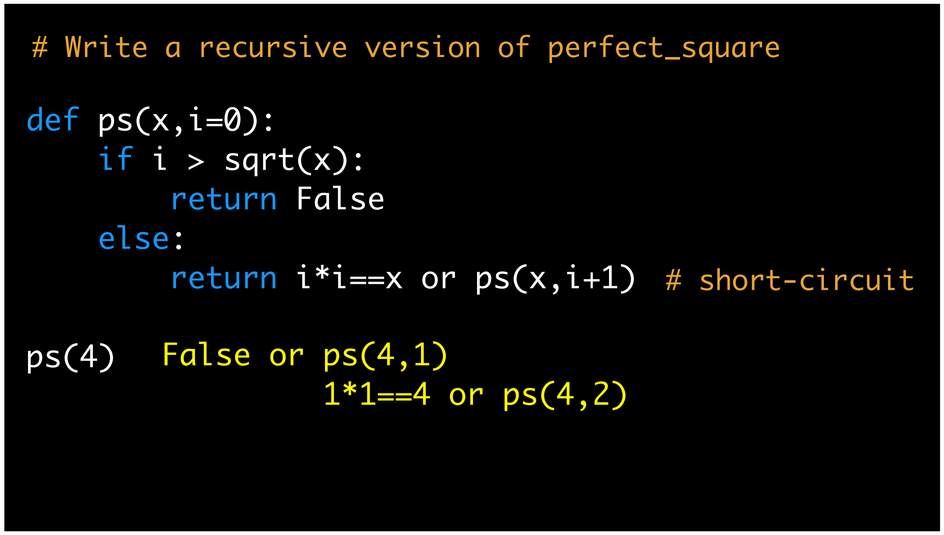
```
# Write a Python function perfect_square that takes a
# single parameter and returns True if this parameter is
# a perfect square and False otherwise

from math import sqrt

def perfect_square(x):
    i = 0
    while i <= sqrt(x):
        if i*i == x:
            return True
        i = i + 1
    return False
```



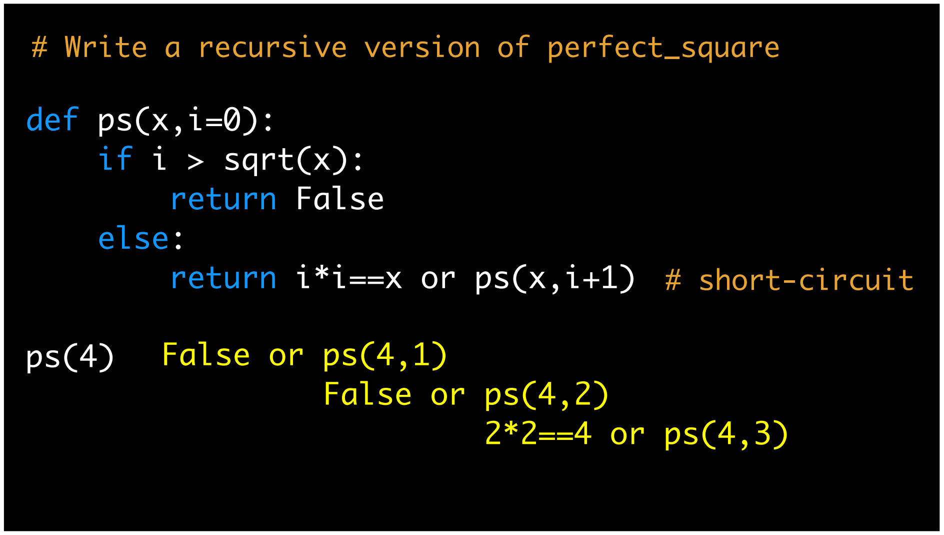
```
# Write a recursive version of perfect_square

def ps(x,i=0):
    if i > sqrt(x):
        return False
    else:
        return i*i==x or ps(x,i+1)  # short-circuit

ps(4)
```



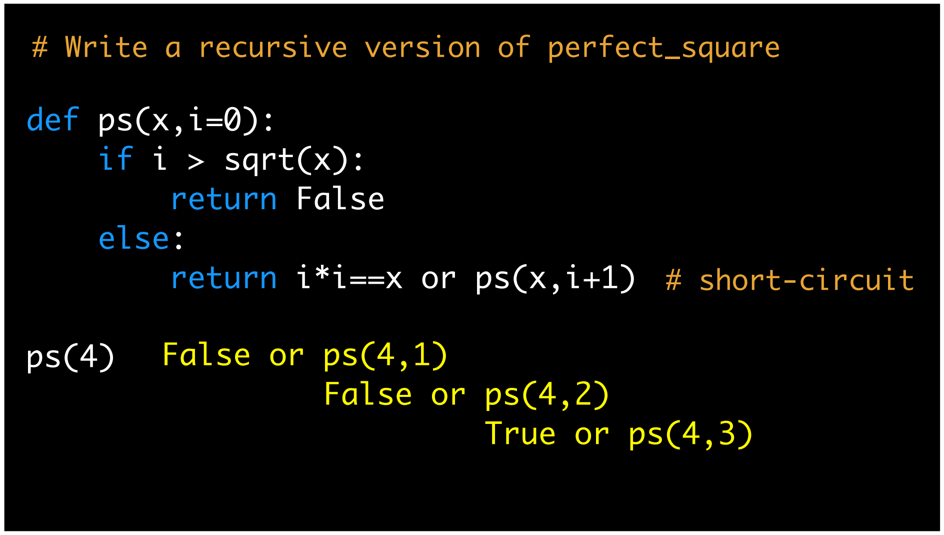
```
# Write a recursive version of perfect_square

def ps(x,i=0):
    if i > sqrt(x):
        return False
    else:
        return i*i==x or ps(x,i+1)  # short-circuit

ps(4)  0*0==4 or ps(4,1)
```



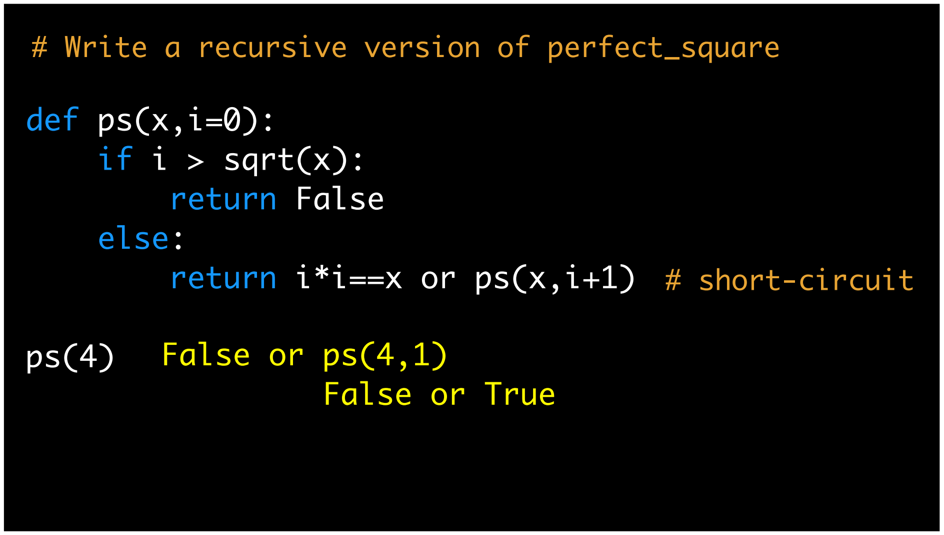
```
# Write a recursive version of perfect_square

def ps(x,i=0):
    if i > sqrt(x):
        return False
    else:
        return i*i==x or ps(x,i+1)  # short-circuit

ps(4)   False or ps(4,1)
                 1*1==4 or ps(4,2)
```



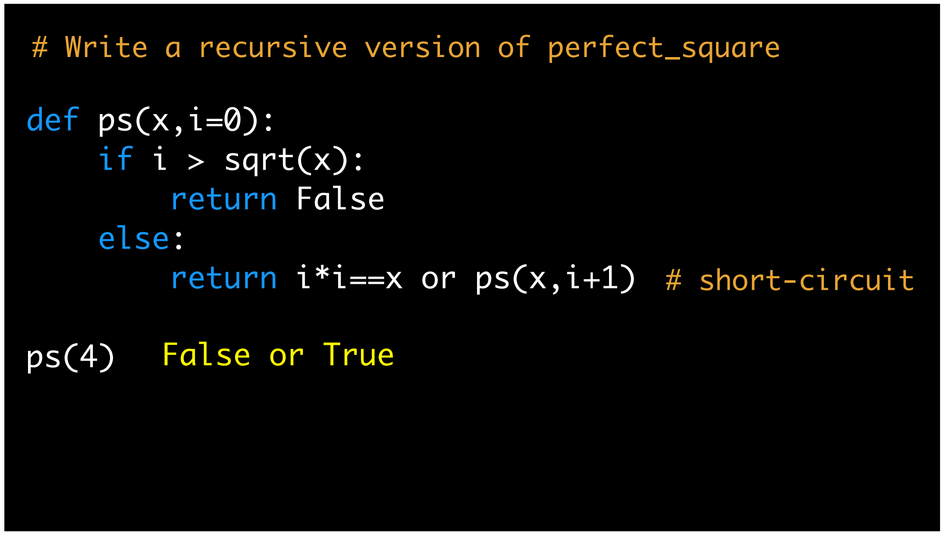
```
# Write a recursive version of perfect_square

def ps(x,i=0):
    if i > sqrt(x):
        return False
    else:
        return i*i==x or ps(x,i+1)  # short-circuit

ps(4)   False or ps(4,1)
                 False or ps(4,2)
                          2*2==4 or ps(4,3)
```



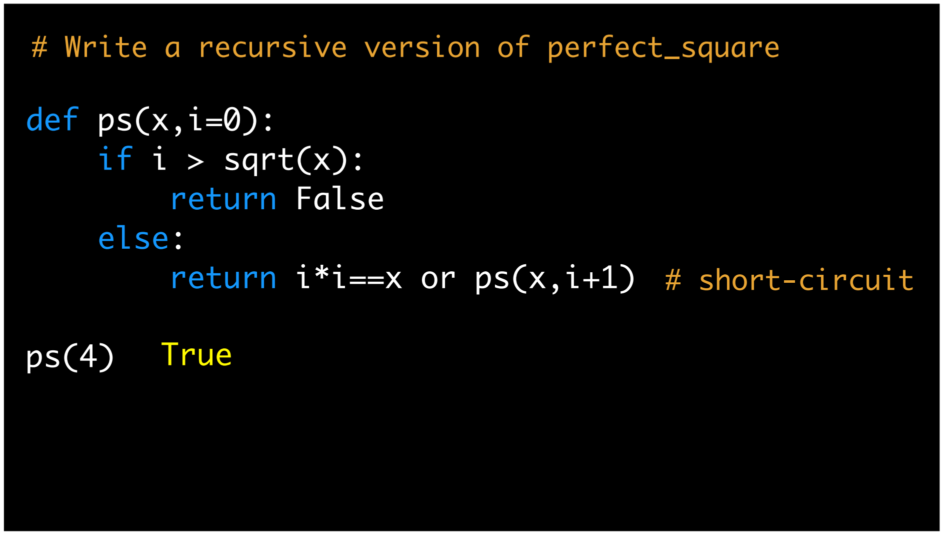
```
# Write a recursive version of perfect_square

def ps(x,i=0):
    if i > sqrt(x):
        return False
    else:
        return i*i==x or ps(x,i+1)  # short-circuit

ps(4)   False or ps(4,1)
                 False or ps(4,2)
                          True or ps(4,3)
```



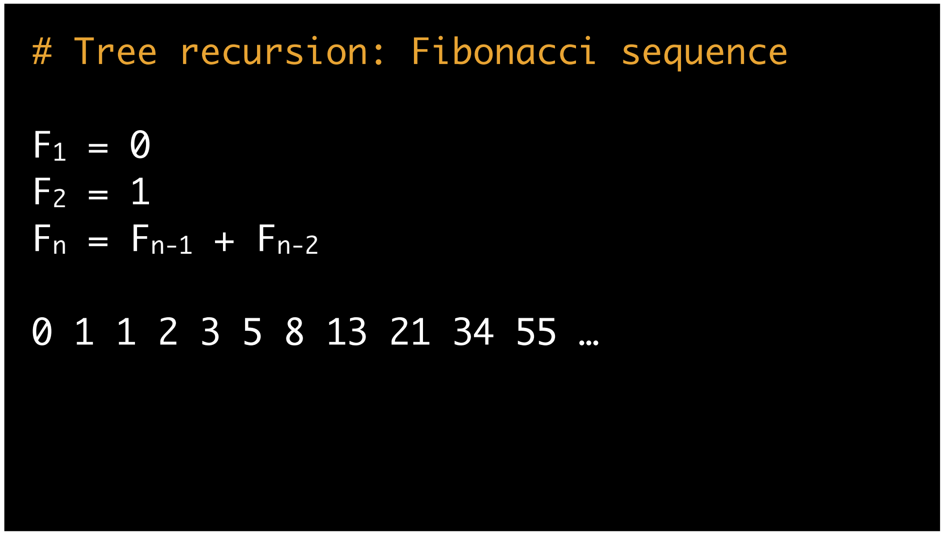
```
# Write a recursive version of perfect_square

def ps(x,i=0):
    if i > sqrt(x):
        return False
    else:
        return i*i==x or ps(x,i+1)  # short-circuit

ps(4)   False or ps(4,1)
                 False or True
```



```
# Write a recursive version of perfect_square

def ps(x,i=0):
    if i > sqrt(x):
        return False
    else:
        return i*i==x or ps(x,i+1)  # short-circuit

ps(4)   False or True
```



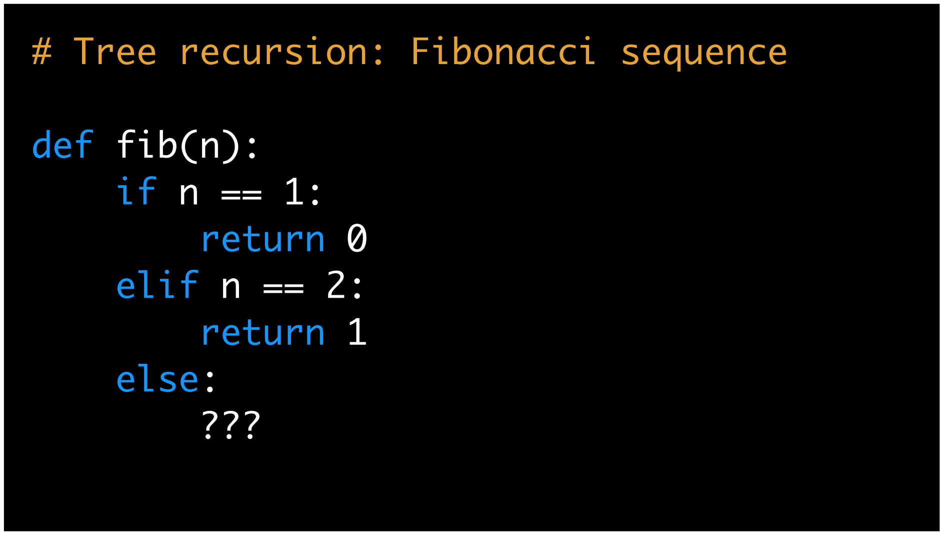
```
# Write a recursive version of perfect_square

def ps(x,i=0):
    if i > sqrt(x):
        return False
    else:
        return i*i==x or ps(x,i+1)  # short-circuit

ps(4)   True
```



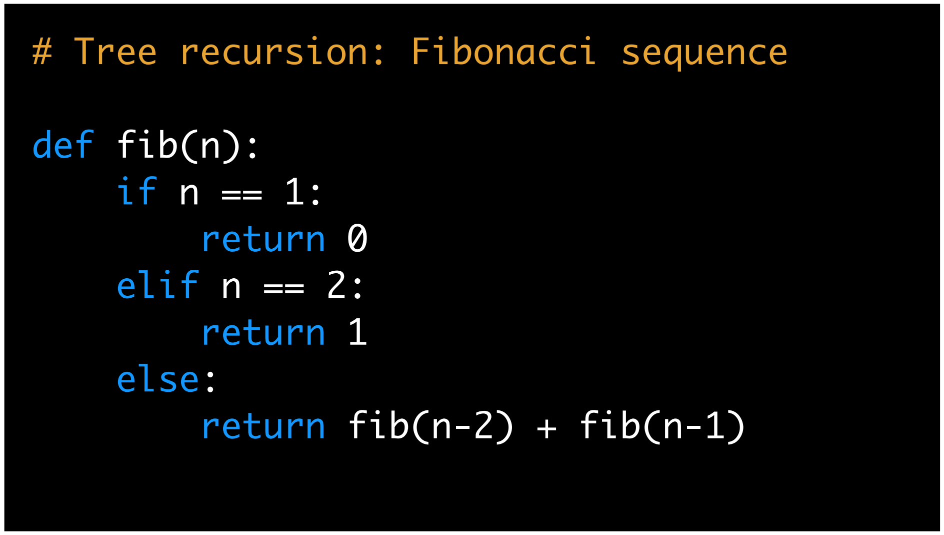
Tree recursion: Fibonacci sequence

$F_1 = 0$
$F_2 = 1$
$F_n = F_{n-1} + F_{n-2}$

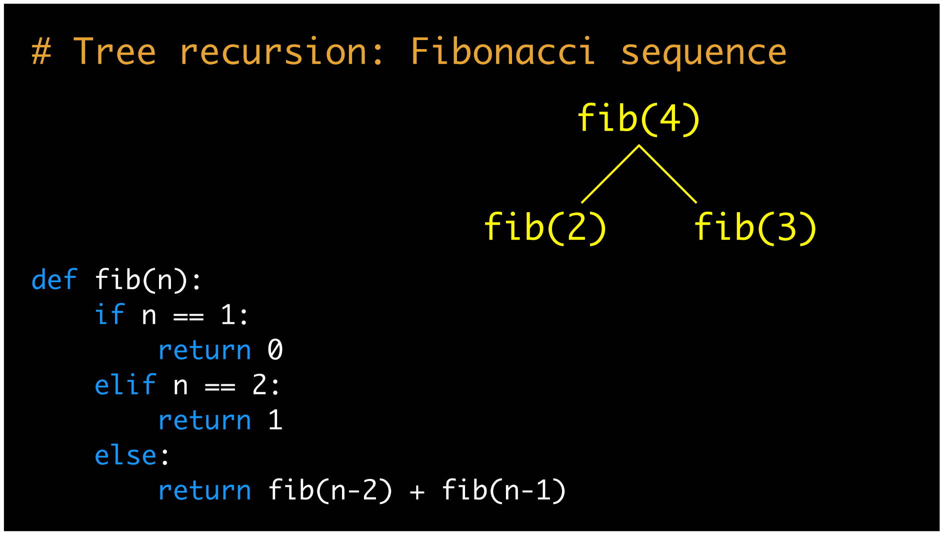
0 1 1 2 3 5 8 13 21 34 55 …



```
# Tree recursion: Fibonacci sequence

def fib(n):
    if n == 1:
        return 0
    elif n == 2:
        return 1
    else:
        ???
```



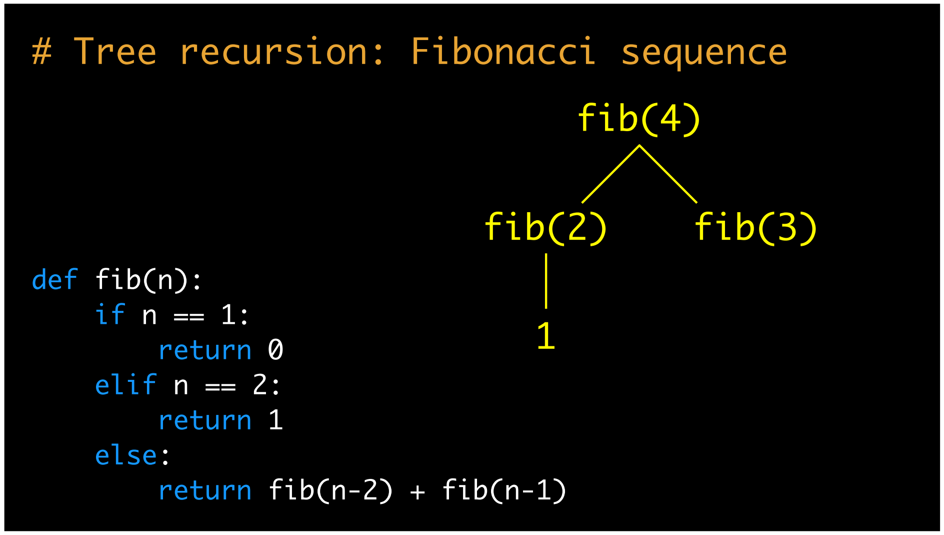
```
# Tree recursion: Fibonacci sequence

def fib(n):
    if n == 1:
        return 0
    elif n == 2:
        return 1
    else:
        return fib(n-2) + fib(n-1)
```



```
# Tree recursion: Fibonacci sequence
```

fib(4)
fib(2)
fib(3)

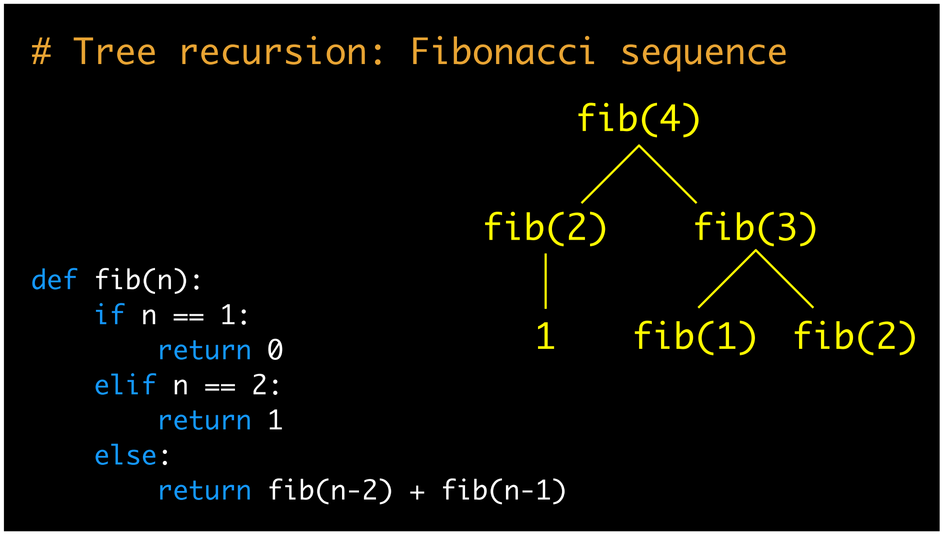
```
def fib(n):
    if n == 1:
        return 0
    elif n == 2:
        return 1
    else:
        return fib(n-2) + fib(n-1)
```



```
# Tree recursion: Fibonacci sequence
```

fib(4)
fib(2)
fib(3)
1

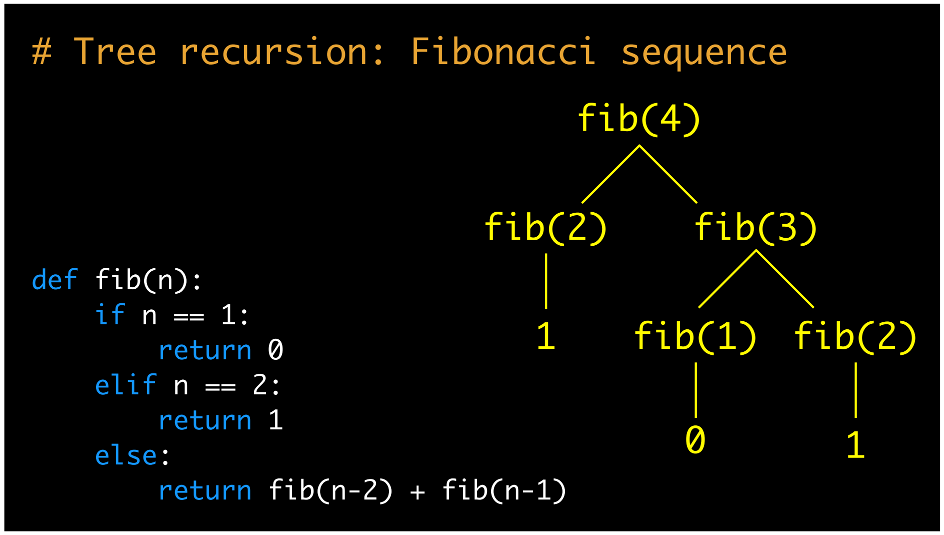
```
def fib(n):
    if n == 1:
        return 0
    elif n == 2:
        return 1
    else:
        return fib(n-2) + fib(n-1)
```



```
# Tree recursion: Fibonacci sequence
```

fib(4)
fib(2) fib(3)
1 fib(1) fib(2)

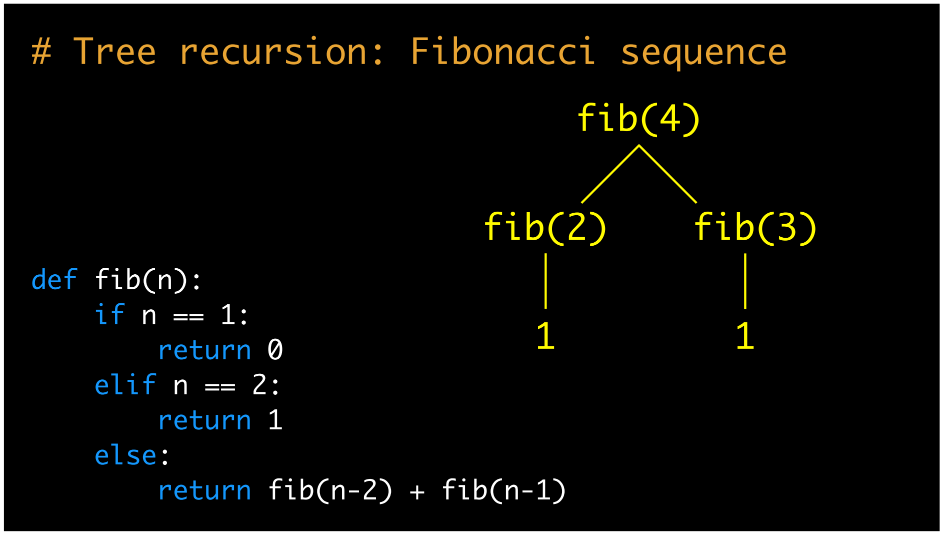
```
def fib(n):
    if n == 1:
        return 0
    elif n == 2:
        return 1
    else:
        return fib(n-2) + fib(n-1)
```



```
# Tree recursion: Fibonacci sequence
```

fib(4)
fib(2) fib(3)
1 fib(1) fib(2)
0 1

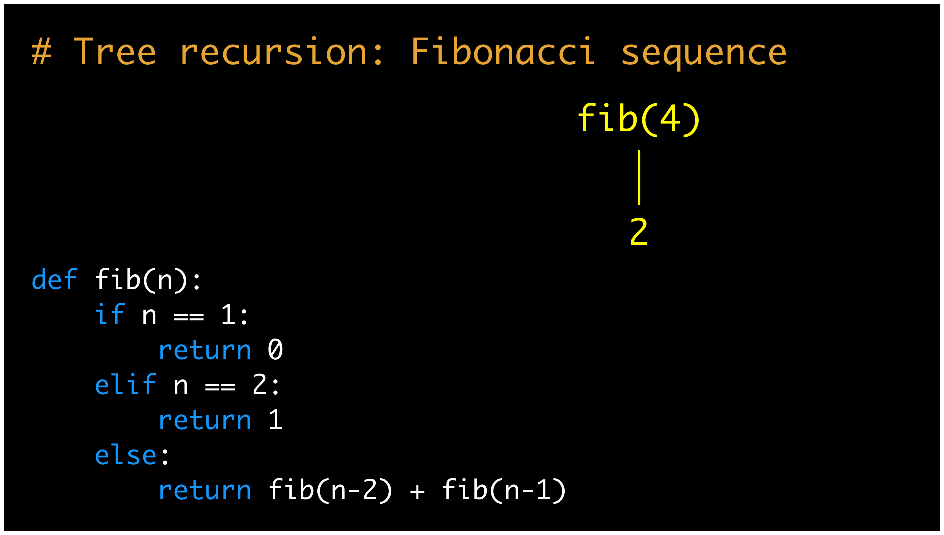
```
def fib(n):
    if n == 1:
        return 0
    elif n == 2:
        return 1
    else:
        return fib(n-2) + fib(n-1)
```



```
# Tree recursion: Fibonacci sequence
```

fib(4)
fib(2) fib(3)
1 1

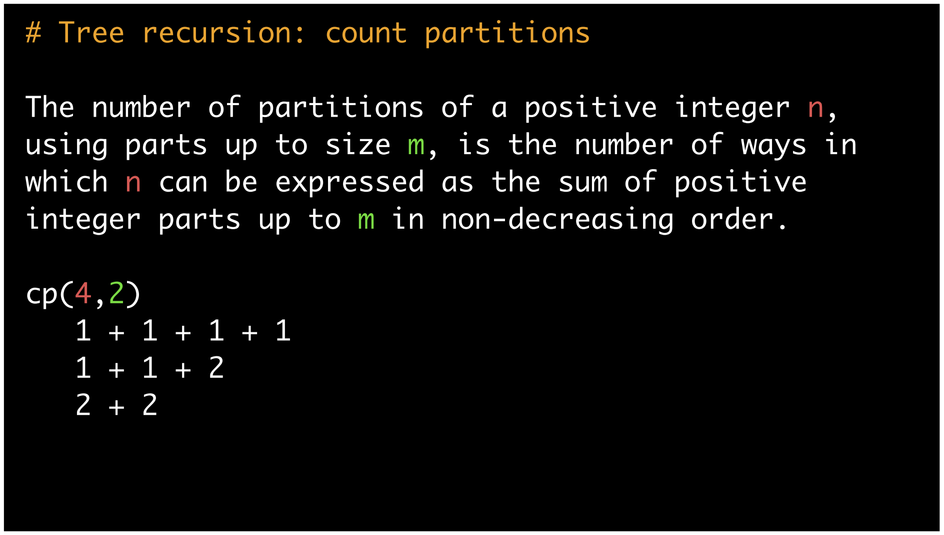
```
def fib(n):
    if n == 1:
        return 0
    elif n == 2:
        return 1
    else:
        return fib(n-2) + fib(n-1)
```



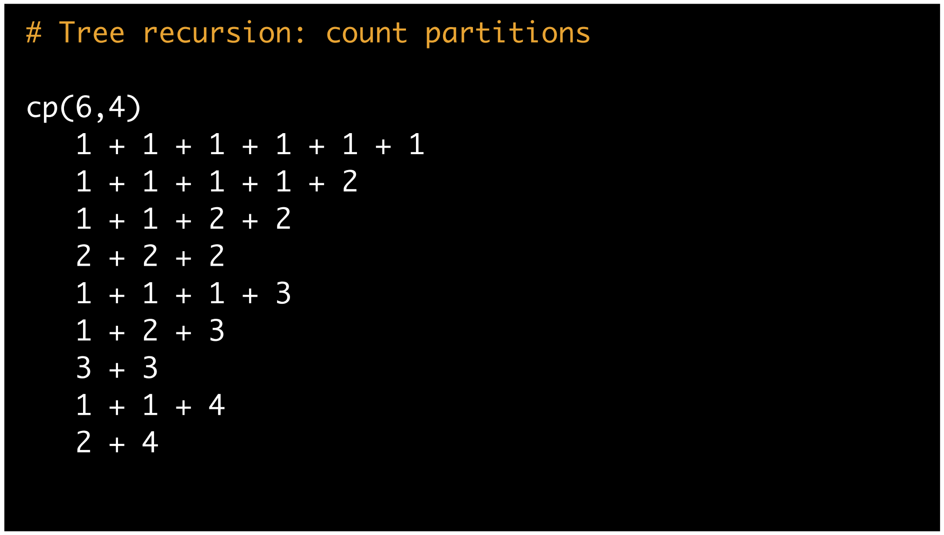
```
# Tree recursion: Fibonacci sequence
```

fib(4)
2

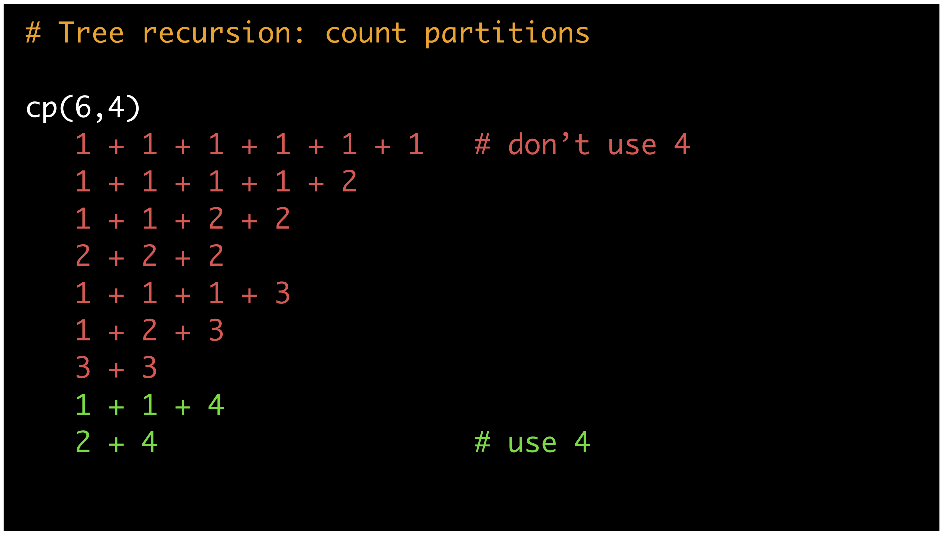
```
def fib(n):
    if n == 1:
        return 0
    elif n == 2:
        return 1
    else:
        return fib(n-2) + fib(n-1)
```



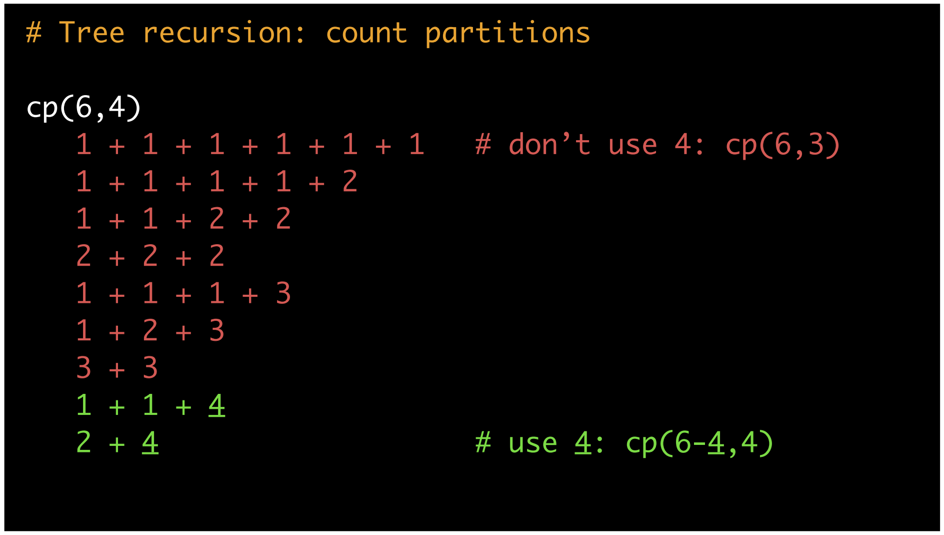
```
# Tree recursion: count partitions

The number of partitions of a positive integer n,
using parts up to size m, is the number of ways in
which n can be expressed as the sum of positive
integer parts up to m in non-decreasing order.

cp(4,2)
   1 + 1 + 1 + 1
   1 + 1 + 2
   2 + 2
```



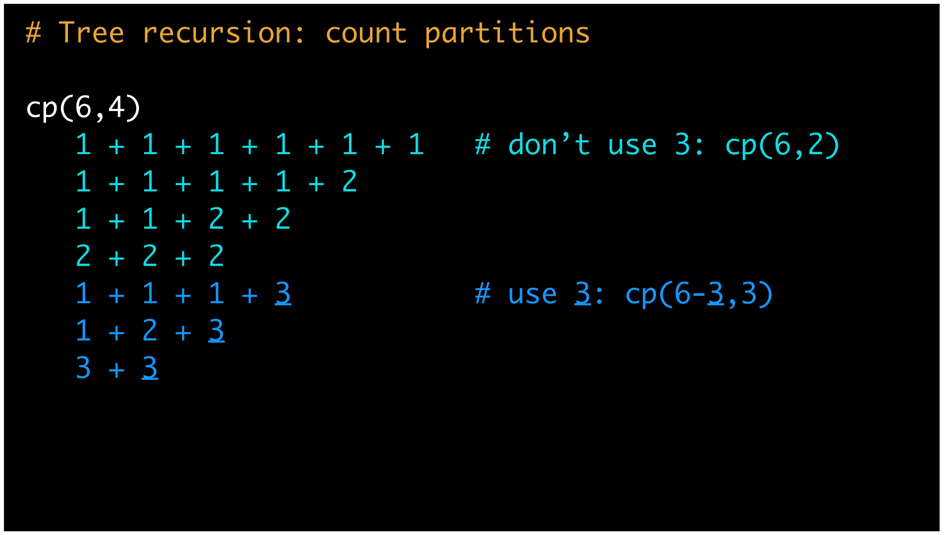
```
# Tree recursion: count partitions

cp(6,4)
   1 + 1 + 1 + 1 + 1 + 1
   1 + 1 + 1 + 1 + 2
   1 + 1 + 2 + 2
   2 + 2 + 2
   1 + 1 + 1 + 3
   1 + 2 + 3
   3 + 3
   1 + 1 + 4
   2 + 4
```



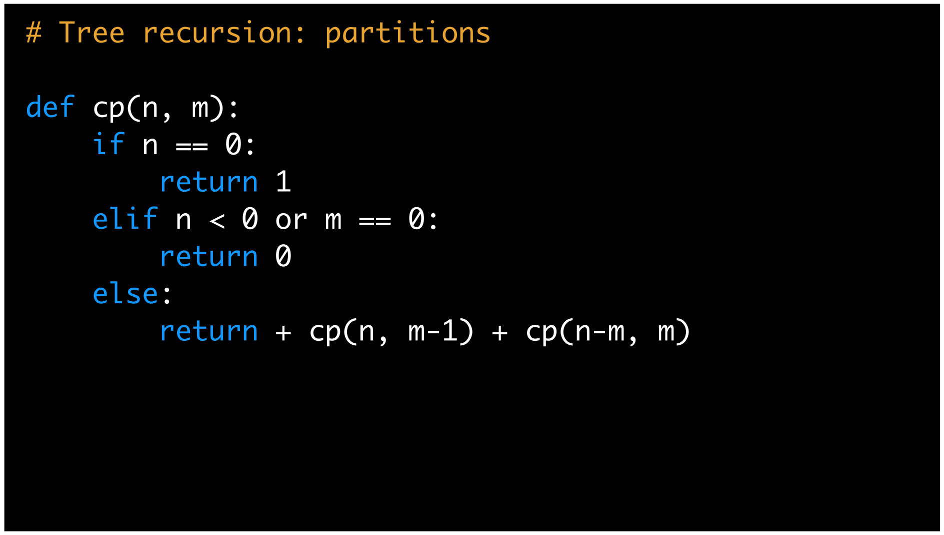
```
# Tree recursion: count partitions

cp(6,4)
   1 + 1 + 1 + 1 + 1 + 1   # don’t use 4
   1 + 1 + 1 + 1 + 2
   1 + 1 + 2 + 2
   2 + 2 + 2
   1 + 1 + 1 + 3
   1 + 2 + 3
   3 + 3
   1 + 1 + 4
   2 + 4                   # use 4
```



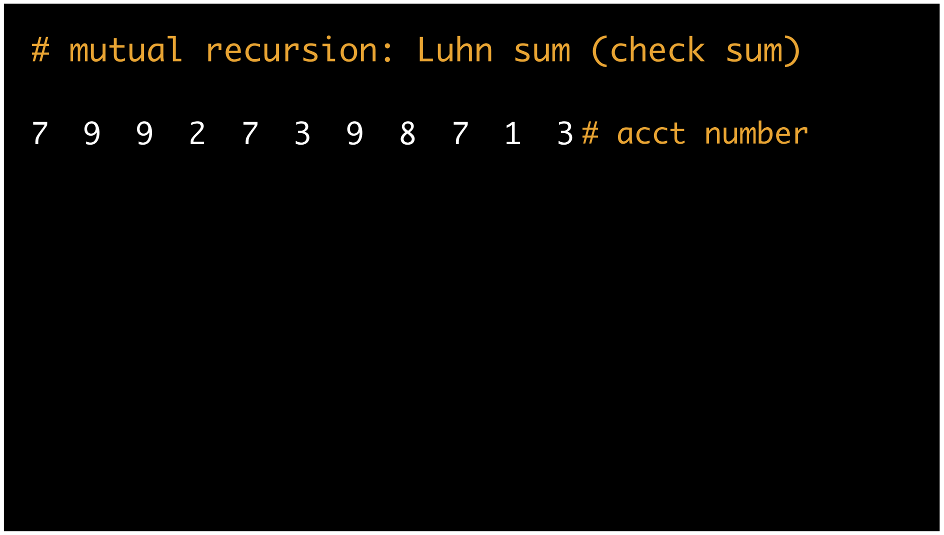
```
# Tree recursion: count partitions

cp(6,4)
   1 + 1 + 1 + 1 + 1 + 1   # don’t use 4: cp(6,3)
   1 + 1 + 1 + 1 + 2
   1 + 1 + 2 + 2
   2 + 2 + 2
   1 + 1 + 1 + 3
   1 + 2 + 3
   3 + 3
   1 + 1 + 4
   2 + 4                   # use 4: cp(6-4,4)
```



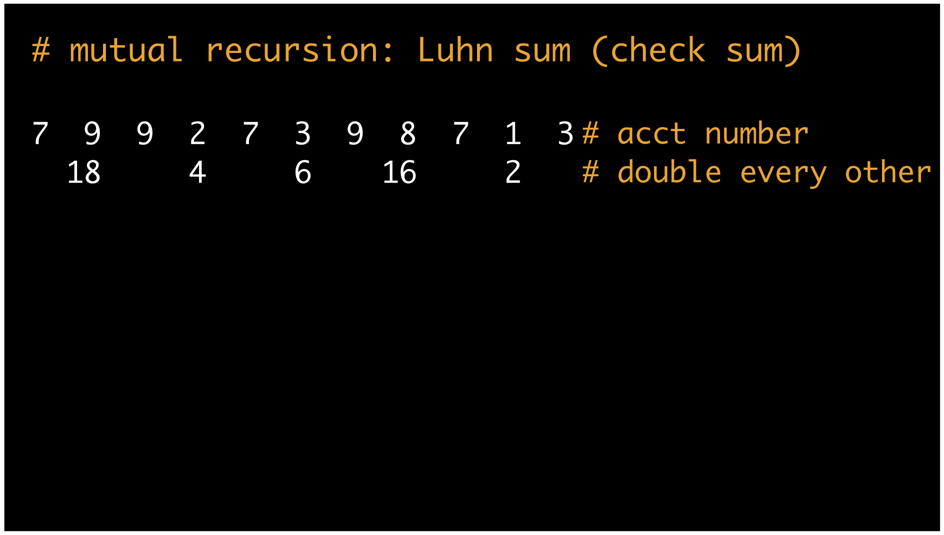
```
# Tree recursion: count partitions

cp(6,4)
   1 + 1 + 1 + 1 + 1 + 1   # don’t use 3: cp(6,2)
   1 + 1 + 1 + 1 + 2
   1 + 1 + 2 + 2
   2 + 2 + 2
   1 + 1 + 1 + 3           # use 3: cp(6-3,3)
   1 + 2 + 3
   3 + 3
```



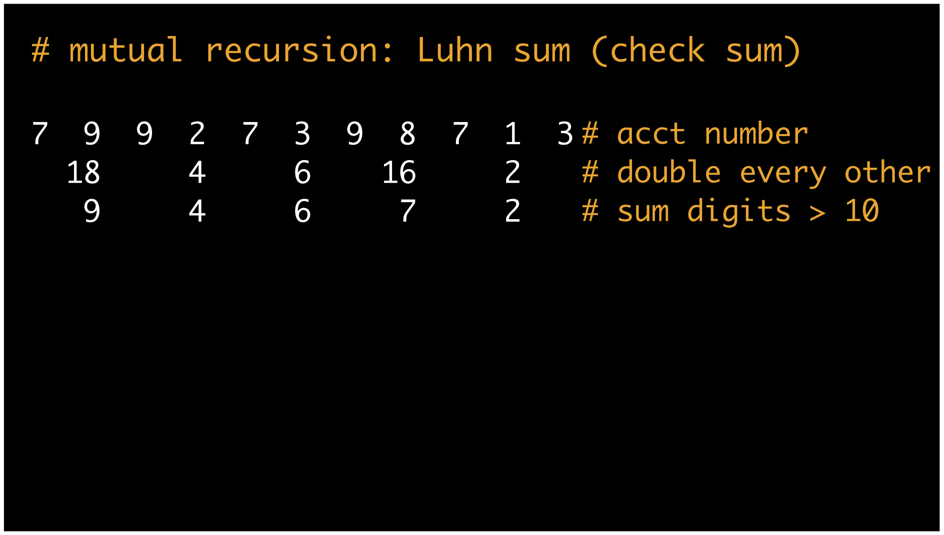
```
# Tree recursion: partitions

def cp(n, m):
    if n == 0:
        return 1
    elif n < 0 or m == 0:
        return 0
    else:
        return + cp(n, m-1) + cp(n-m, m)
```



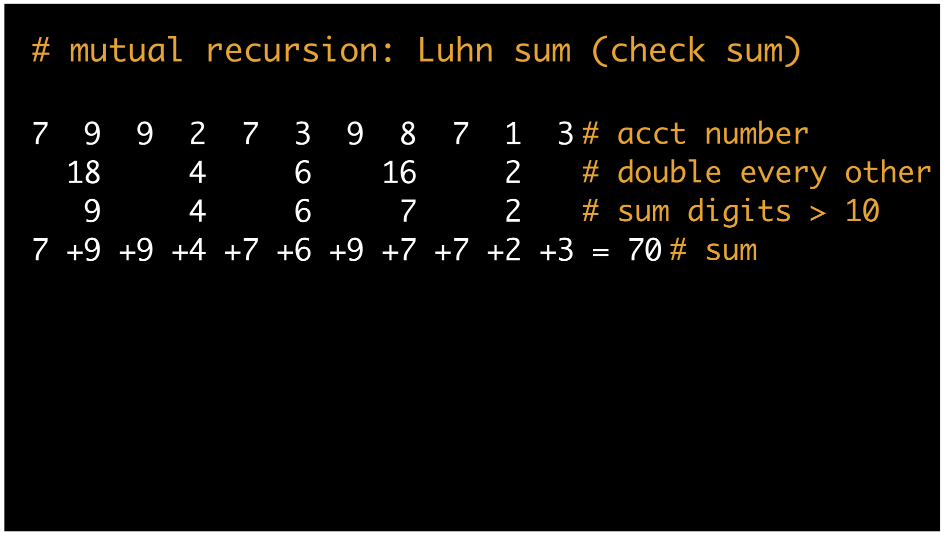
```
# mutual recursion: Luhn sum (check sum)

7  9  9  2  7  3  9  8  7  1  3# acct number
```



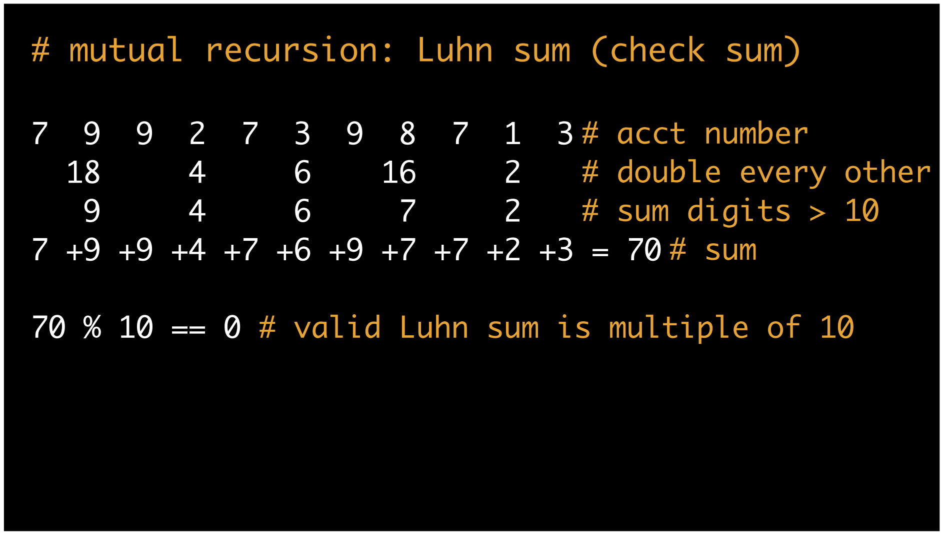
```
# mutual recursion: Luhn sum (check sum)

7  9  9  2  7  3  9  8  7  1  3# acct number
  18     4     6    16     2    # double every other
```



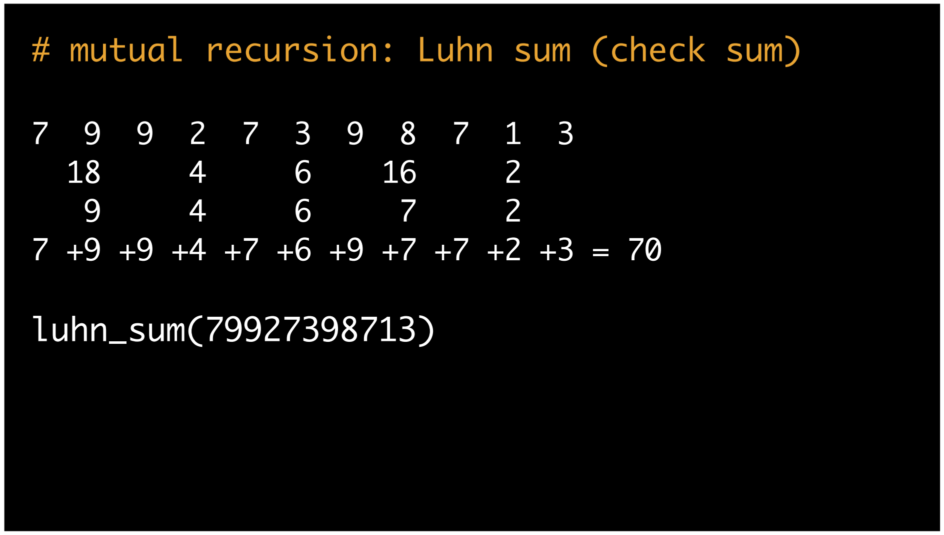
```
# mutual recursion: Luhn sum (check sum)

7  9  9  2  7  3  9  8  7  1  3# acct number
  18    4     6    16     2    # double every other
   9    4     6     7     2    # sum digits > 10
```



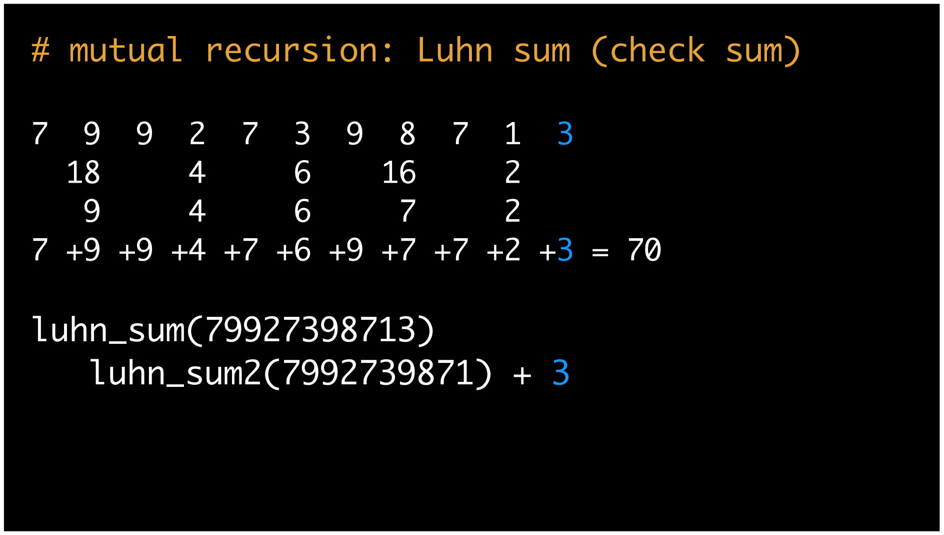
```
# mutual recursion: Luhn sum (check sum)

7  9  9  2  7  3  9  8  7  1  3# acct number
  18    4     6    16     2    # double every other
   9    4     6     7     2    # sum digits > 10
7 +9 +9 +4 +7 +6 +9 +7 +7 +2 +3 = 70# sum
```



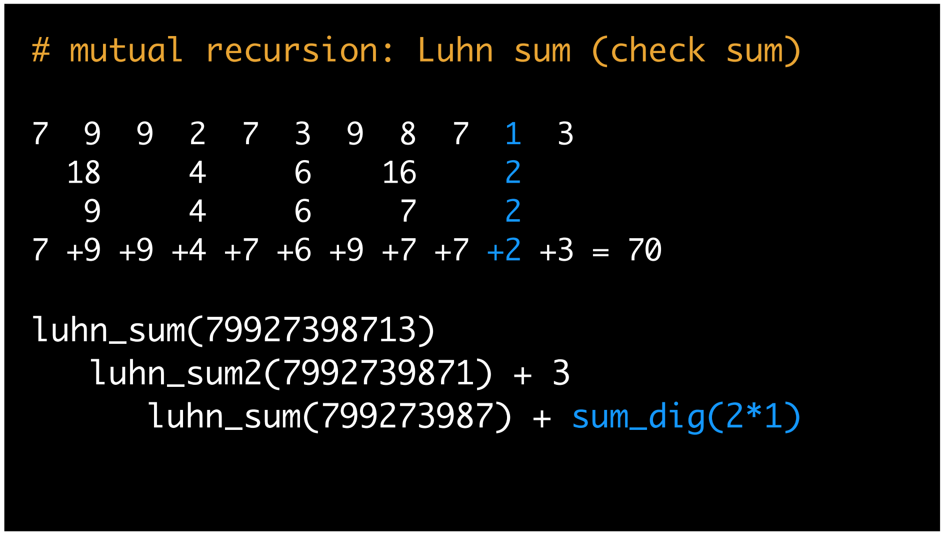
```
# mutual recursion: Luhn sum (check sum)

7  9  9  2  7  3  9  8  7  1  3# acct number
  18    4     6    16     2    # double every other
   9    4     6     7     2    # sum digits > 10
7 +9 +9 +4 +7 +6 +9 +7 +7 +2 +3 = 70# sum

70 % 10 == 0 # valid Luhn sum is multiple of 10
```



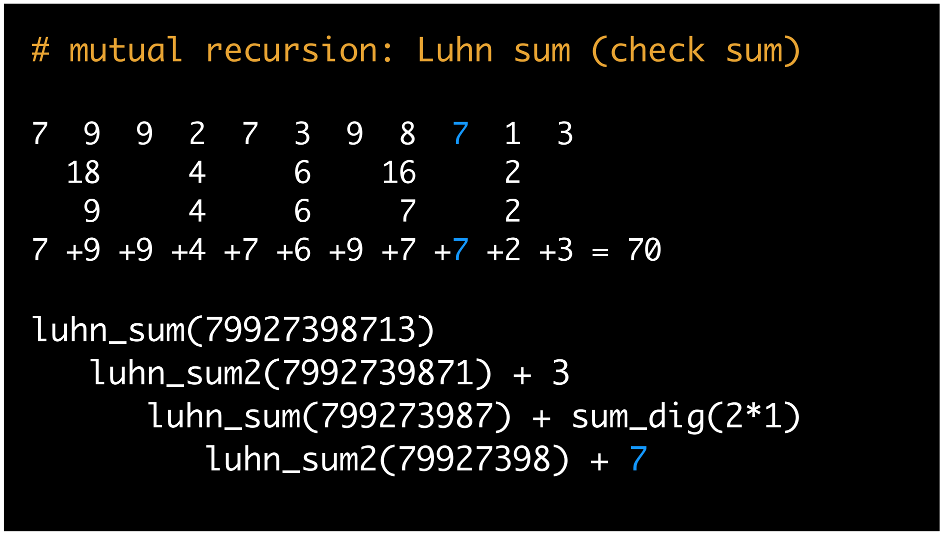
```
# mutual recursion: Luhn sum (check sum)

7  9  9  2  7  3  9  8  7  1  3
  18    4     6    16     2
   9    4     6     7     2
7 +9 +9 +4 +7 +6 +9 +7 +7 +2 +3 = 70

luhn_sum(79927398713)
```



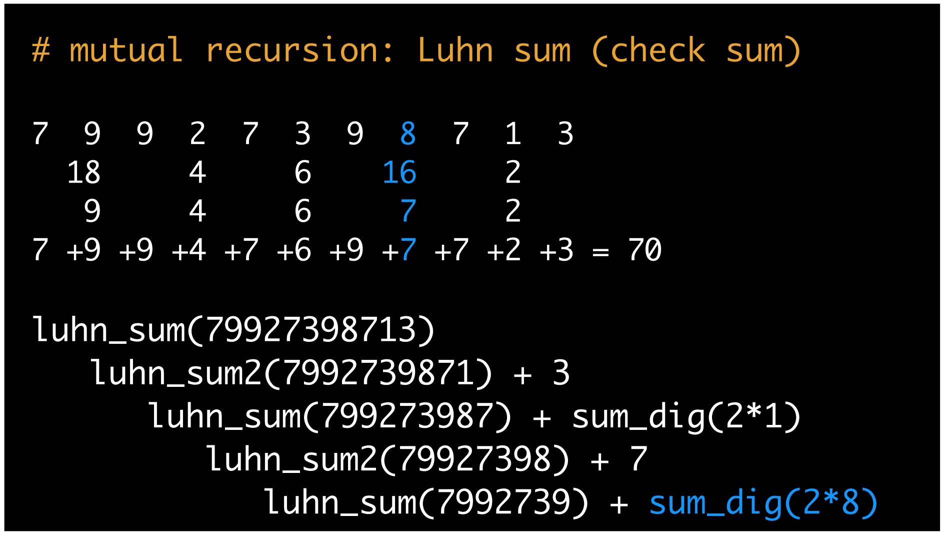
```
# mutual recursion: Luhn sum (check sum)

7  9  9  2  7  3  9  8  7  1  3
  18     4     6    16     2
   9     4     6     7     2
7 +9 +9 +4 +7 +6 +9 +7 +7 +2 +3 = 70

luhn_sum(79927398713)
   luhn_sum2(7992739871) + 3
```



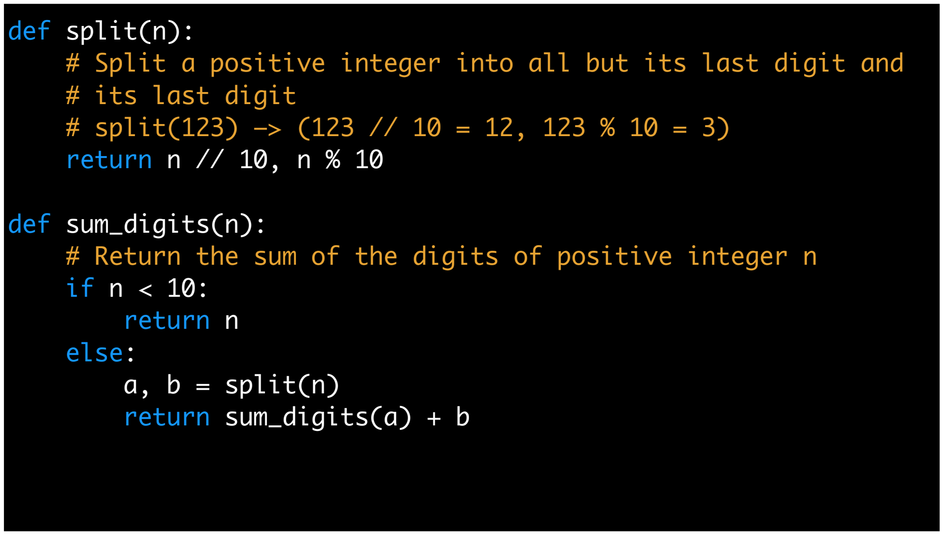
```
# mutual recursion: Luhn sum (check sum)

7  9  9  2  7  3  9  8  7  1  3
  18     4     6    16     2
   9     4     6     7     2
7 +9 +9 +4 +7 +6 +9 +7 +7 +2 +3 = 70

luhn_sum(79927398713)
   luhn_sum2(7992739871) + 3
      luhn_sum(799273987) + sum_dig(2*1)
```



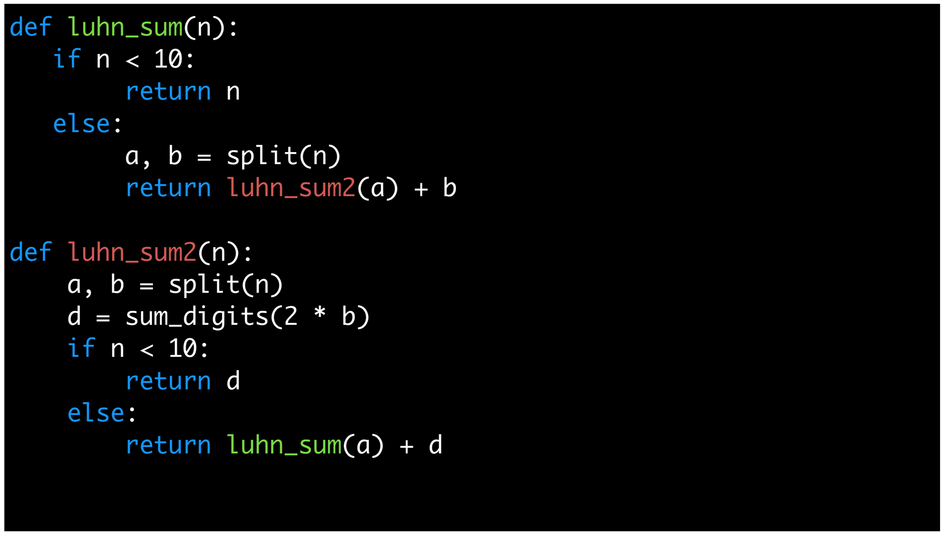
```
# mutual recursion: Luhn sum (check sum)

7  9  9  2  7  3  9  8  7  1  3
  18     4     6    16     2
   9     4     6     7     2
7 +9 +9 +4 +7 +6 +9 +7 +7 +2 +3 = 70

luhn_sum(79927398713)
   luhn_sum2(7992739871) + 3
      luhn_sum(799273987) + sum_dig(2*1)
         luhn_sum2(79927398) + 7
```



```
# mutual recursion: Luhn sum (check sum)

7  9  9  2  7  3  9  8  7  1  3
  18     4     6    16     2
   9     4     6     7     2
7 +9 +9 +4 +7 +6 +9 +7 +7 +2 +3 = 70

luhn_sum(79927398713)
   luhn_sum2(7992739871) + 3
      luhn_sum(799273987) + sum_dig(2*1)
         luhn_sum2(79927398) + 7
            luhn_sum(7992739) + sum_dig(2*8)
```



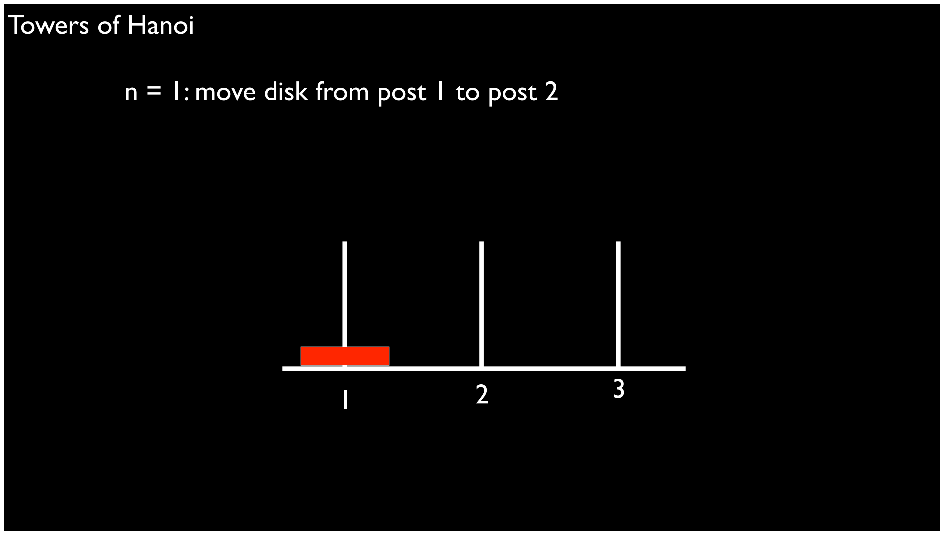
```python
def split(n):
    # Split a positive integer into all but its last digit and
    # its last digit
    # split(123) -> (123 // 10 = 12, 123 % 10 = 3)
    return n // 10, n % 10

def sum_digits(n):
    # Return the sum of the digits of positive integer n
    if n < 10:
        return n
    else:
        a, b = split(n)
        return sum_digits(a) + b
```



```python
def luhn_sum(n):
   if n < 10:
        return n
   else:
        a, b = split(n)
        return luhn_sum2(a) + b

def luhn_sum2(n):
    a, b = split(n)
    d = sum_digits(2 * b)
    if n < 10:
        return d
    else:
        return luhn_sum(a) + d
```

Towers of Hanoi

http://haubergs.com/hanoi

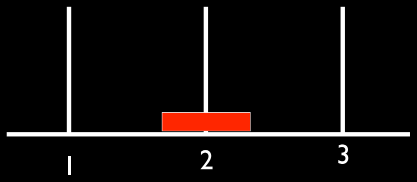


Towers of Hanoi

n = 1: move disk from post 1 to post 2

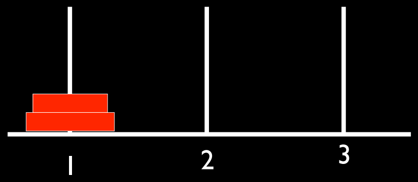
Towers of Hanoi

n = 1: move disk from post 1 to post 2



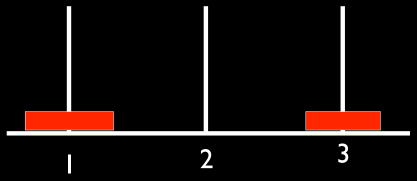
Towers of Hanoi

n = 2: move disks from post 1 to post 2



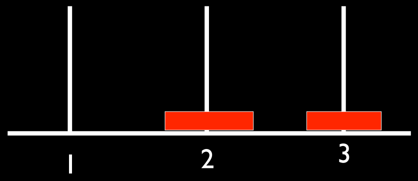
Towers of Hanoi

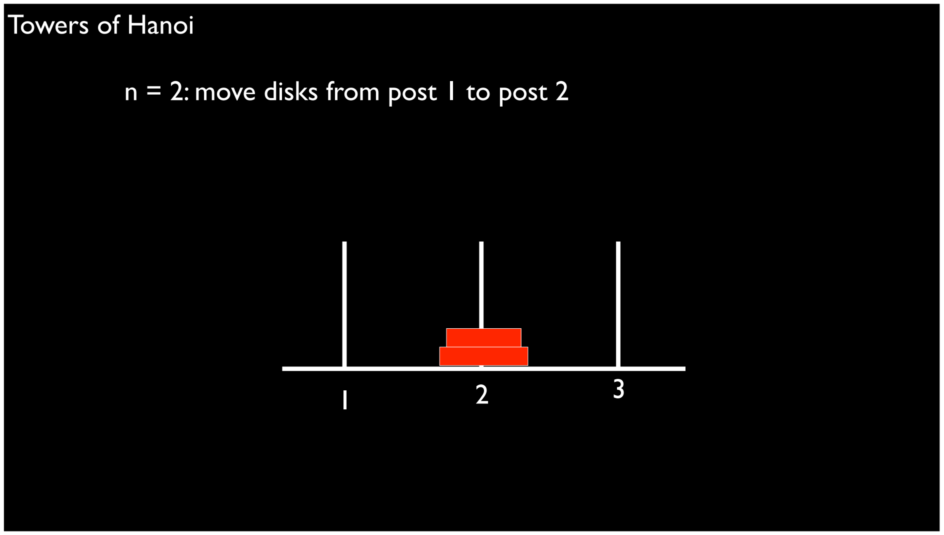
n = 2: move disks from post 1 to post 2



Towers of Hanoi

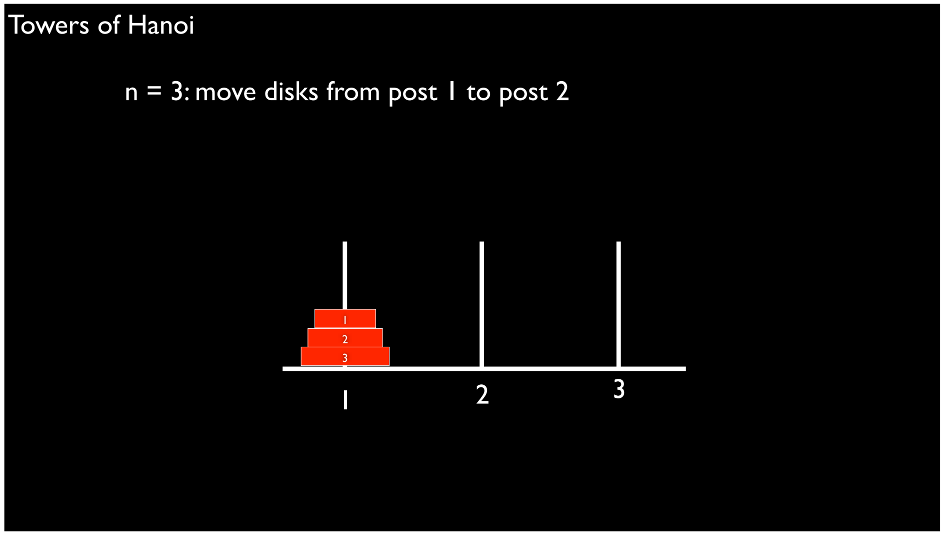
n = 2: move disks from post 1 to post 2





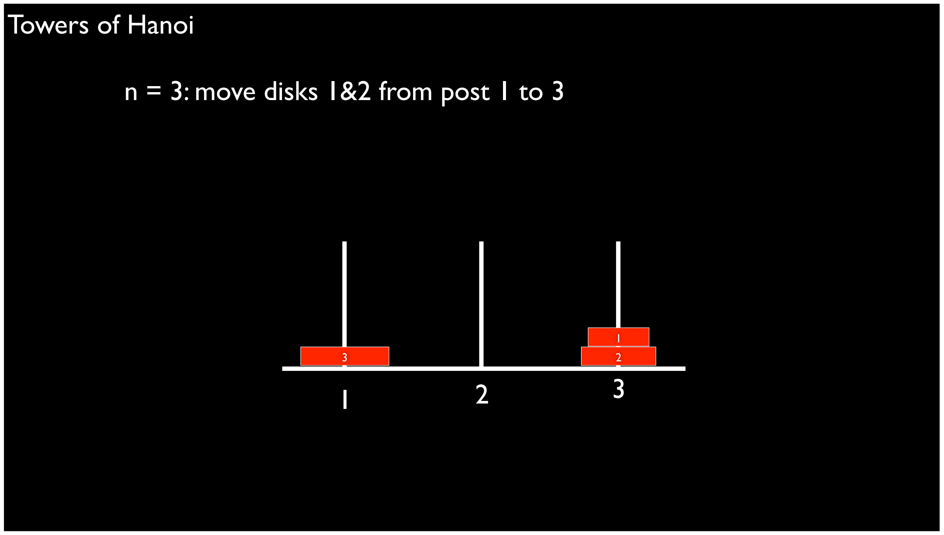
Towers of Hanoi

n = 2: move disks from post 1 to post 2



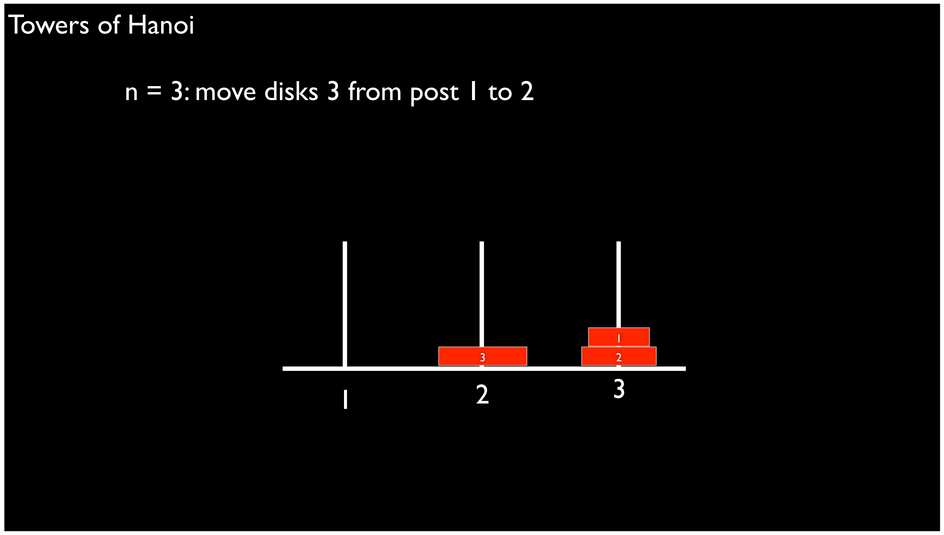
Towers of Hanoi

n = 3: move disks from post 1 to post 2



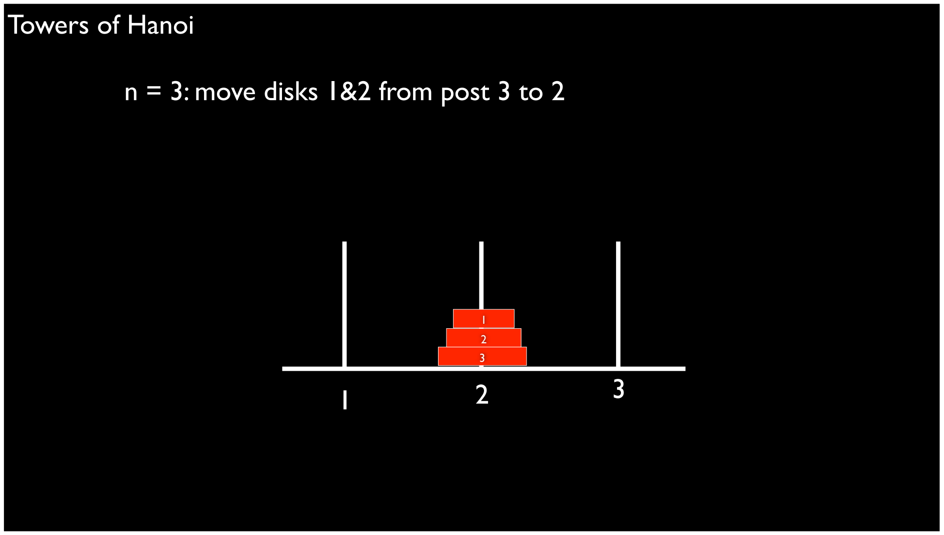
Towers of Hanoi

n = 3: move disks 1&2 from post 1 to 3



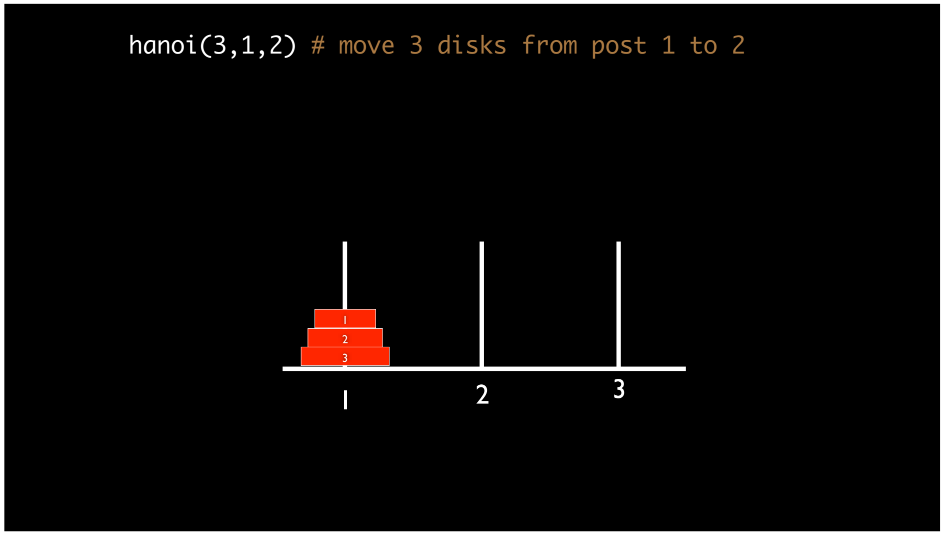
Towers of Hanoi

n = 3: move disks 3 from post 1 to 2

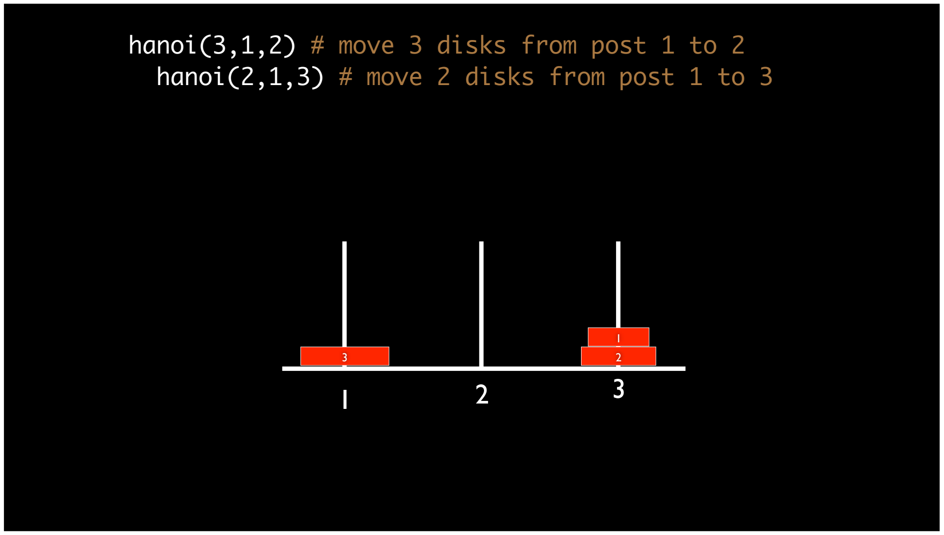


Towers of Hanoi

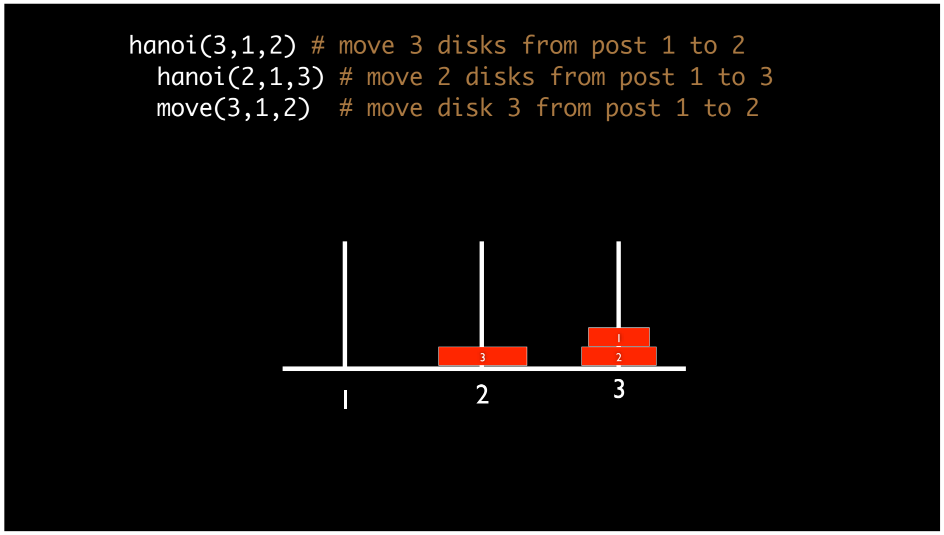
n = 3: move disks 1&2 from post 3 to 2



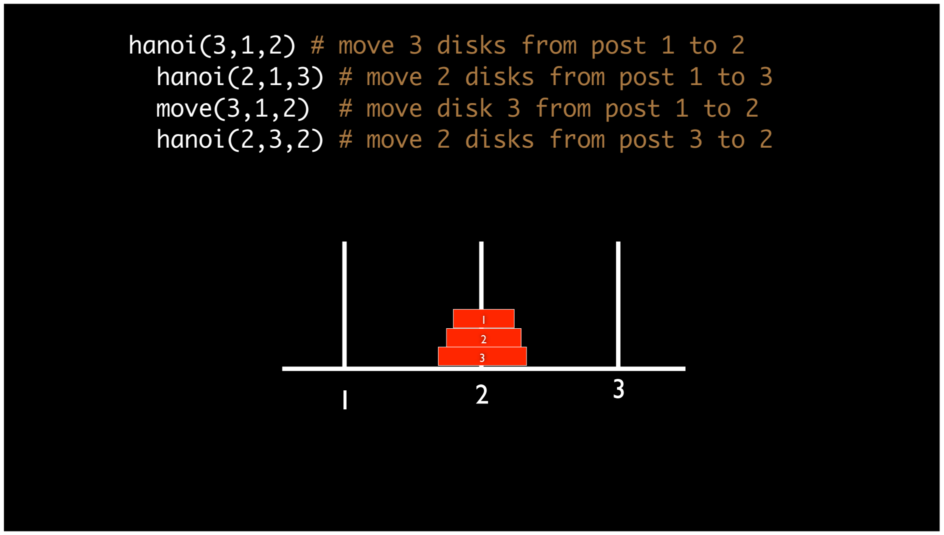
```
hanoi(3,1,2) # move 3 disks from post 1 to 2
```



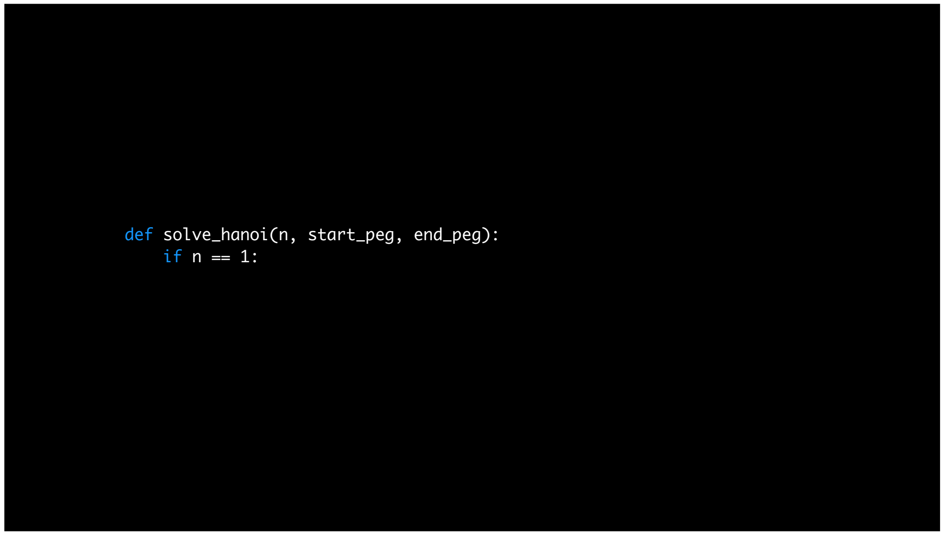
```
hanoi(3,1,2) # move 3 disks from post 1 to 2
  hanoi(2,1,3) # move 2 disks from post 1 to 3
```



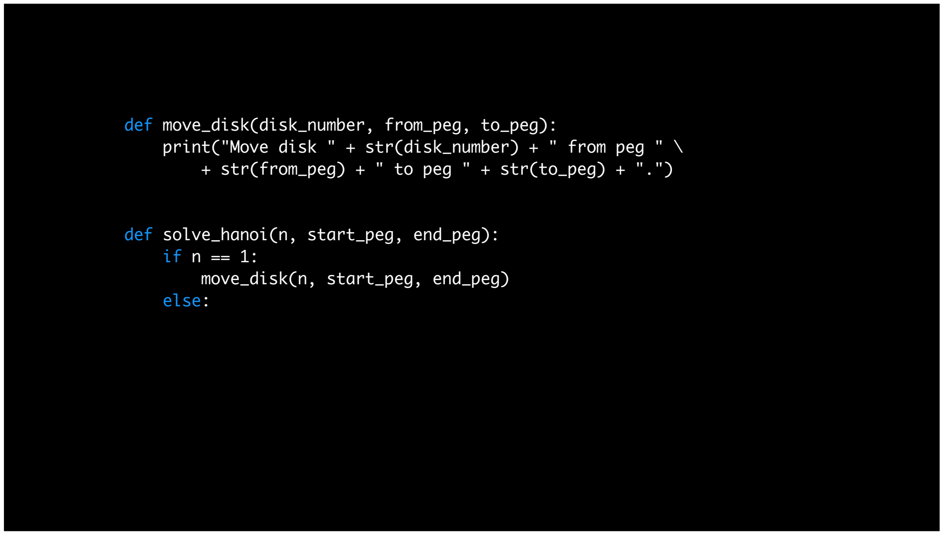
```
hanoi(3,1,2) # move 3 disks from post 1 to 2
  hanoi(2,1,3) # move 2 disks from post 1 to 3
  move(3,1,2)  # move disk 3 from post 1 to 2
```



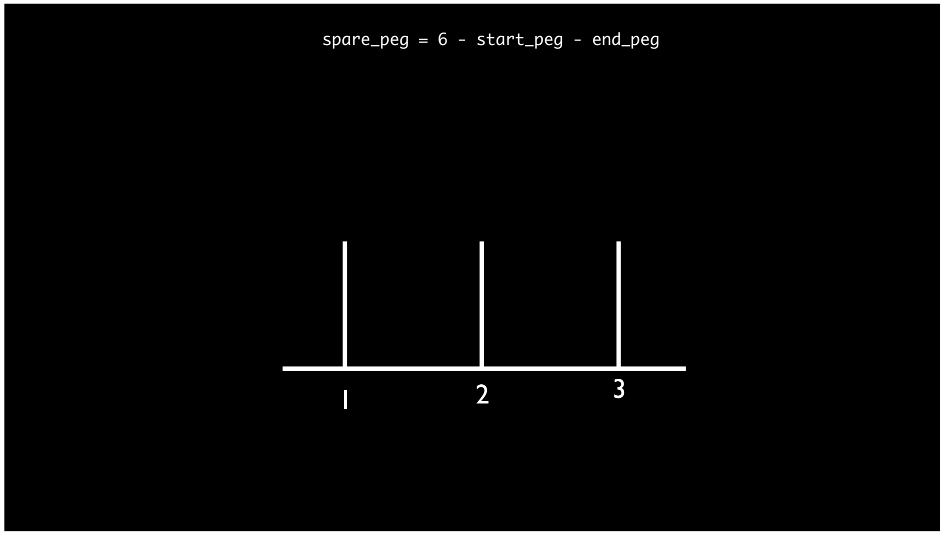
```
hanoi(3,1,2) # move 3 disks from post 1 to 2
  hanoi(2,1,3) # move 2 disks from post 1 to 3
  move(3,1,2)  # move disk 3 from post 1 to 2
  hanoi(2,3,2) # move 2 disks from post 3 to 2
```



```
def solve_hanoi(n, start_peg, end_peg):
    if n == 1:
```



```
def move_disk(disk_number, from_peg, to_peg):
    print("Move disk " + str(disk_number) + " from peg " \
        + str(from_peg) + " to peg " + str(to_peg) + ".")


def solve_hanoi(n, start_peg, end_peg):
    if n == 1:
        move_disk(n, start_peg, end_peg)
    else:
```



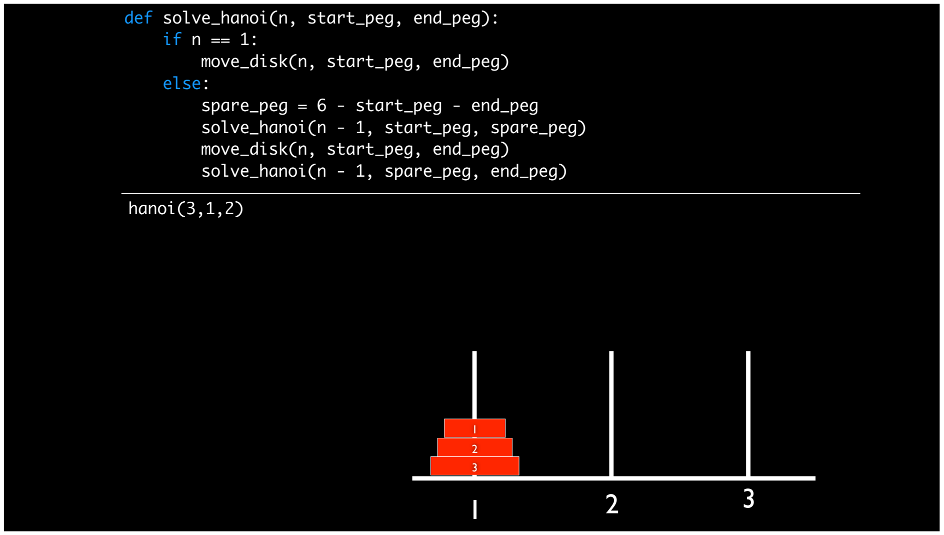
```
spare_peg = 6 - start_peg - end_peg
```

```
def move_disk(disk_number, from_peg, to_peg):
    print("Move disk " + str(disk_number) + " from peg " \
        + str(from_peg) + " to peg " + str(to_peg) + ".")


def solve_hanoi(n, start_peg, end_peg):
    if n == 1:
        move_disk(n, start_peg, end_peg)
    else:
```

```
def move_disk(disk_number, from_peg, to_peg):
    print("Move disk " + str(disk_number) + " from peg " \
        + str(from_peg) + " to peg " + str(to_peg) + ".")


def solve_hanoi(n, start_peg, end_peg):
    if n == 1:
        move_disk(n, start_peg, end_peg)
    else:
        spare_peg = 6 - start_peg - end_peg
        solve_hanoi(n - 1, start_peg, spare_peg)
```

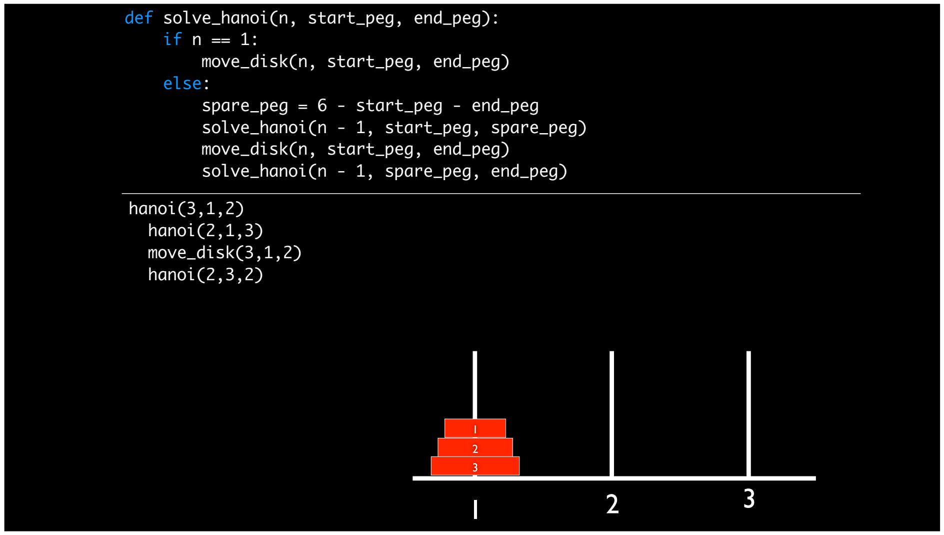
```
def move_disk(disk_number, from_peg, to_peg):
    print("Move disk " + str(disk_number) + " from peg " \
        + str(from_peg) + " to peg " + str(to_peg) + ".")


def solve_hanoi(n, start_peg, end_peg):
    if n == 1:
        move_disk(n, start_peg, end_peg)
    else:
        spare_peg = 6 - start_peg - end_peg
        solve_hanoi(n - 1, start_peg, spare_peg)
        move_disk(n, start_peg, end_peg)
```

```
def move_disk(disk_number, from_peg, to_peg):
    print("Move disk " + str(disk_number) + " from peg " \
        + str(from_peg) + " to peg " + str(to_peg) + ".")


def solve_hanoi(n, start_peg, end_peg):
    if n == 1:
        move_disk(n, start_peg, end_peg)
    else:
        spare_peg = 6 - start_peg - end_peg
        solve_hanoi(n - 1, start_peg, spare_peg)
        move_disk(n, start_peg, end_peg)
        solve_hanoi(n - 1, spare_peg, end_peg)
```



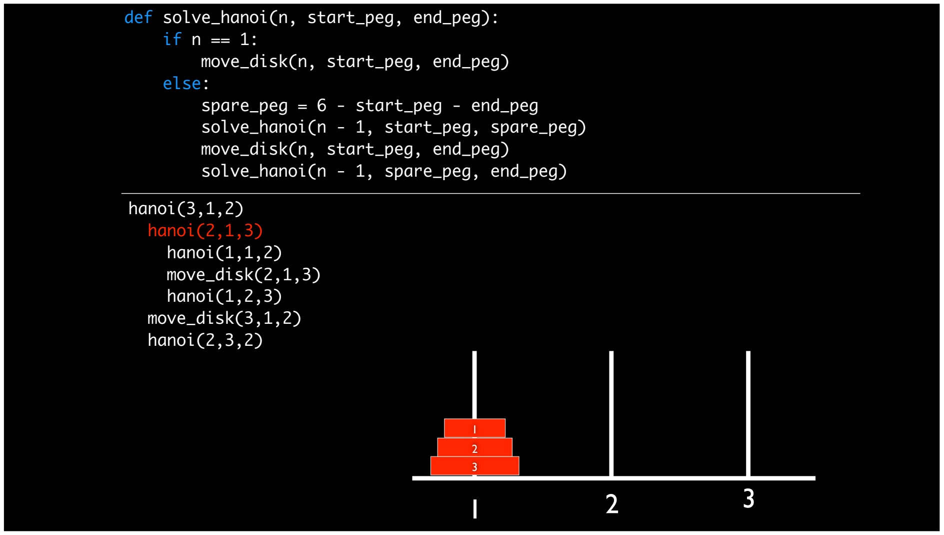
```
def solve_hanoi(n, start_peg, end_peg):
    if n == 1:
        move_disk(n, start_peg, end_peg)
    else:
        spare_peg = 6 - start_peg - end_peg
        solve_hanoi(n - 1, start_peg, spare_peg)
        move_disk(n, start_peg, end_peg)
        solve_hanoi(n - 1, spare_peg, end_peg)
```

```
hanoi(3,1,2)
```



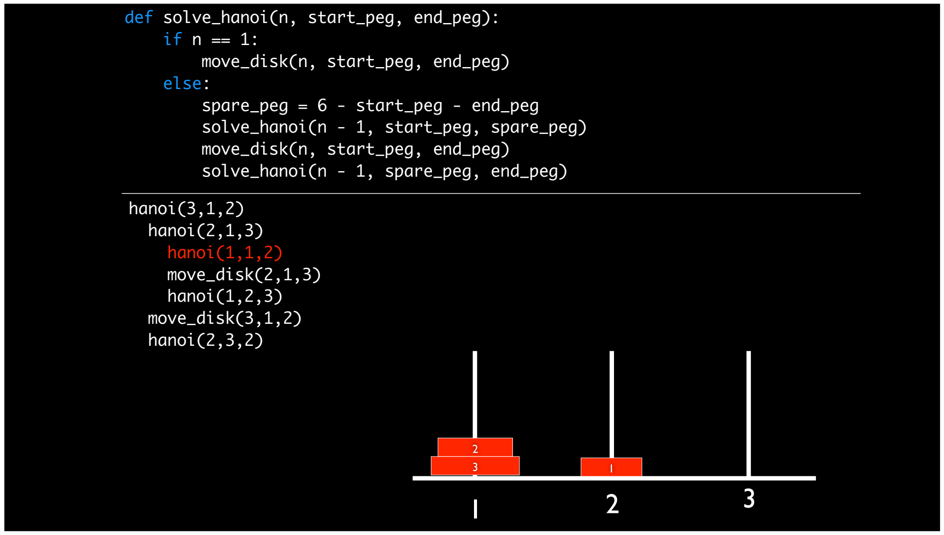
```
def solve_hanoi(n, start_peg, end_peg):
    if n == 1:
        move_disk(n, start_peg, end_peg)
    else:
        spare_peg = 6 - start_peg - end_peg
        solve_hanoi(n - 1, start_peg, spare_peg)
        move_disk(n, start_peg, end_peg)
        solve_hanoi(n - 1, spare_peg, end_peg)
```

```
hanoi(3,1,2)
  hanoi(2,1,3)
  move_disk(3,1,2)
  hanoi(2,3,2)
```



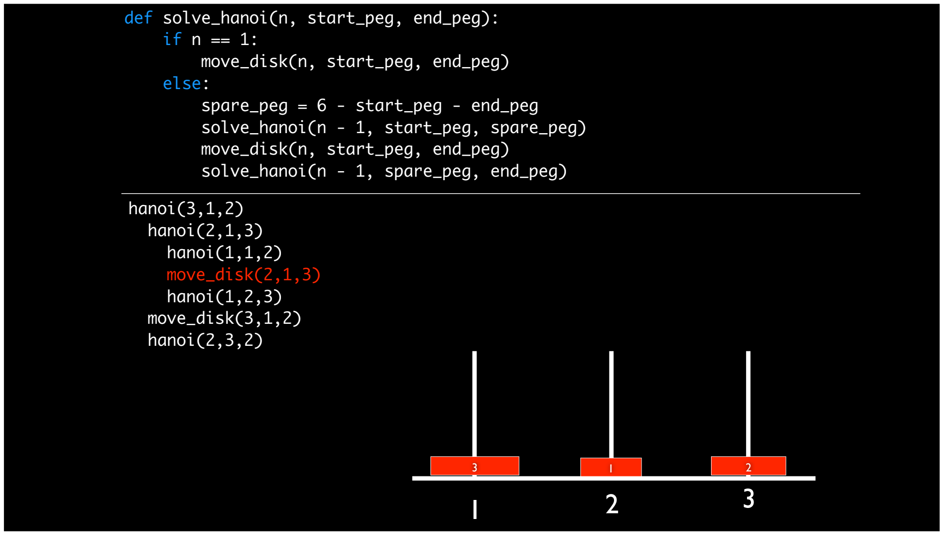
```
def solve_hanoi(n, start_peg, end_peg):
    if n == 1:
        move_disk(n, start_peg, end_peg)
    else:
        spare_peg = 6 - start_peg - end_peg
        solve_hanoi(n - 1, start_peg, spare_peg)
        move_disk(n, start_peg, end_peg)
        solve_hanoi(n - 1, spare_peg, end_peg)
```

```
hanoi(3,1,2)
  hanoi(2,1,3)
    hanoi(1,1,2)
    move_disk(2,1,3)
    hanoi(1,2,3)
  move_disk(3,1,2)
  hanoi(2,3,2)
```



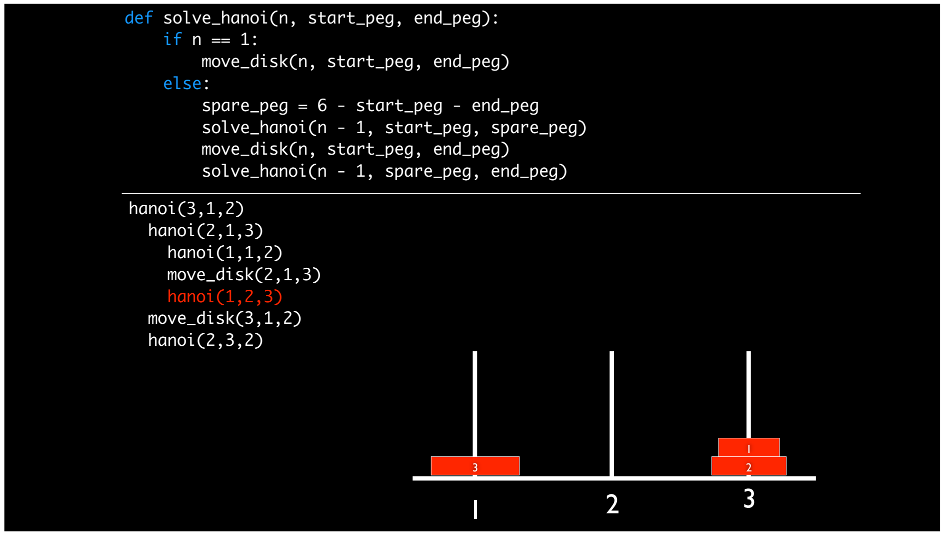
```
def solve_hanoi(n, start_peg, end_peg):
    if n == 1:
        move_disk(n, start_peg, end_peg)
    else:
        spare_peg = 6 - start_peg - end_peg
        solve_hanoi(n - 1, start_peg, spare_peg)
        move_disk(n, start_peg, end_peg)
        solve_hanoi(n - 1, spare_peg, end_peg)
```

```
hanoi(3,1,2)
  hanoi(2,1,3)
    hanoi(1,1,2)
    move_disk(2,1,3)
    hanoi(1,2,3)
  move_disk(3,1,2)
  hanoi(2,3,2)
```



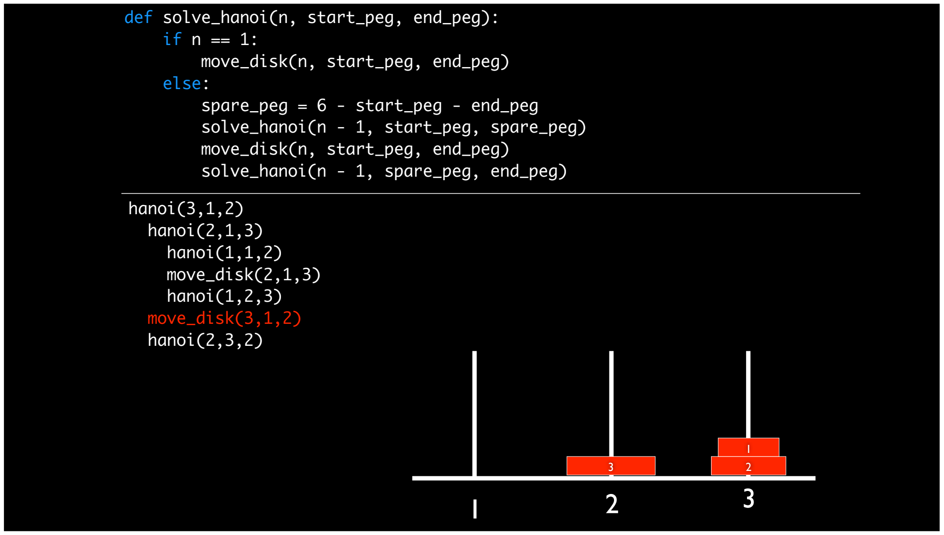
```
def solve_hanoi(n, start_peg, end_peg):
    if n == 1:
        move_disk(n, start_peg, end_peg)
    else:
        spare_peg = 6 - start_peg - end_peg
        solve_hanoi(n - 1, start_peg, spare_peg)
        move_disk(n, start_peg, end_peg)
        solve_hanoi(n - 1, spare_peg, end_peg)
```

```
hanoi(3,1,2)
  hanoi(2,1,3)
    hanoi(1,1,2)
    move_disk(2,1,3)
    hanoi(1,2,3)
  move_disk(3,1,2)
  hanoi(2,3,2)
```



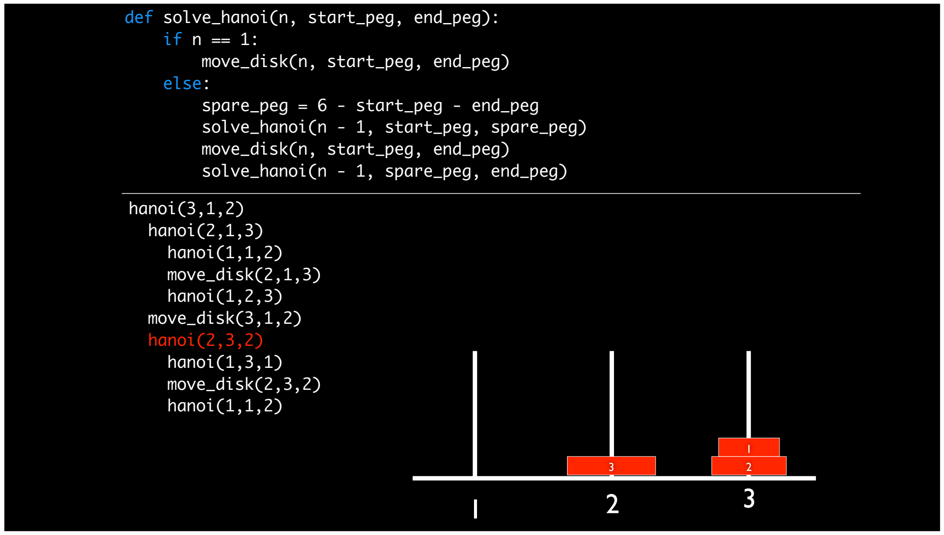
```
def solve_hanoi(n, start_peg, end_peg):
    if n == 1:
        move_disk(n, start_peg, end_peg)
    else:
        spare_peg = 6 - start_peg - end_peg
        solve_hanoi(n - 1, start_peg, spare_peg)
        move_disk(n, start_peg, end_peg)
        solve_hanoi(n - 1, spare_peg, end_peg)
```

```
hanoi(3,1,2)
  hanoi(2,1,3)
    hanoi(1,1,2)
    move_disk(2,1,3)
    hanoi(1,2,3)
  move_disk(3,1,2)
  hanoi(2,3,2)
```



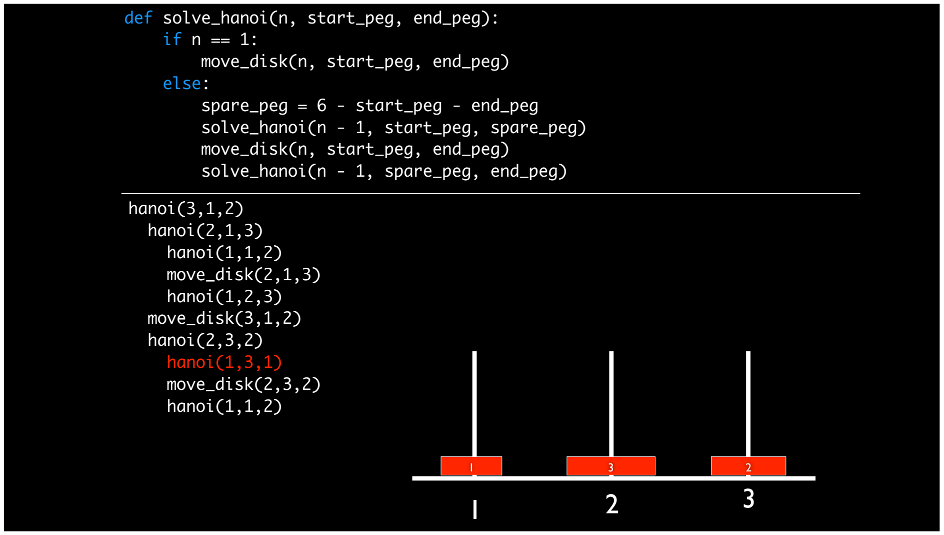
```
def solve_hanoi(n, start_peg, end_peg):
    if n == 1:
        move_disk(n, start_peg, end_peg)
    else:
        spare_peg = 6 - start_peg - end_peg
        solve_hanoi(n - 1, start_peg, spare_peg)
        move_disk(n, start_peg, end_peg)
        solve_hanoi(n - 1, spare_peg, end_peg)
```

```
hanoi(3,1,2)
  hanoi(2,1,3)
    hanoi(1,1,2)
    move_disk(2,1,3)
    hanoi(1,2,3)
  move_disk(3,1,2)
  hanoi(2,3,2)
```



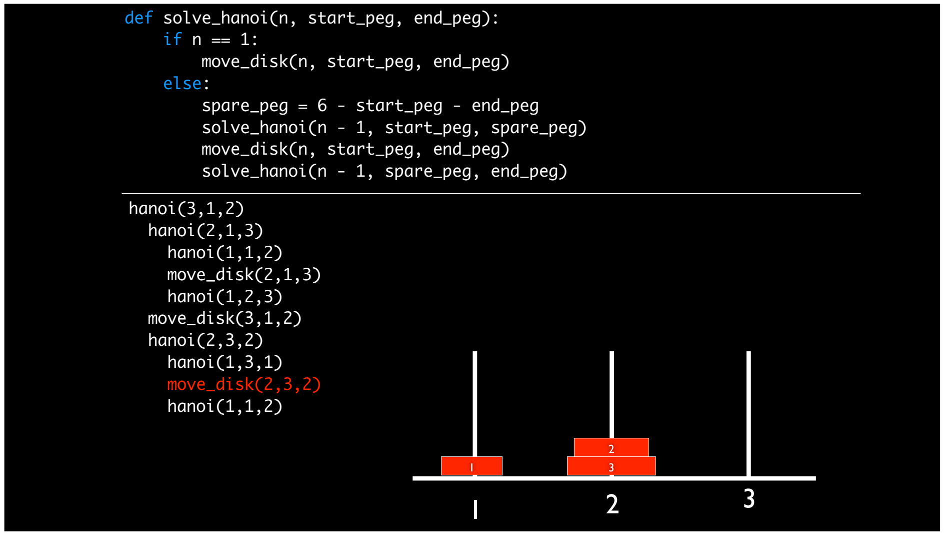
```
def solve_hanoi(n, start_peg, end_peg):
    if n == 1:
        move_disk(n, start_peg, end_peg)
    else:
        spare_peg = 6 - start_peg - end_peg
        solve_hanoi(n - 1, start_peg, spare_peg)
        move_disk(n, start_peg, end_peg)
        solve_hanoi(n - 1, spare_peg, end_peg)
```

```
hanoi(3,1,2)
  hanoi(2,1,3)
    hanoi(1,1,2)
    move_disk(2,1,3)
    hanoi(1,2,3)
  move_disk(3,1,2)
  hanoi(2,3,2)
    hanoi(1,3,1)
    move_disk(2,3,2)
    hanoi(1,1,2)
```



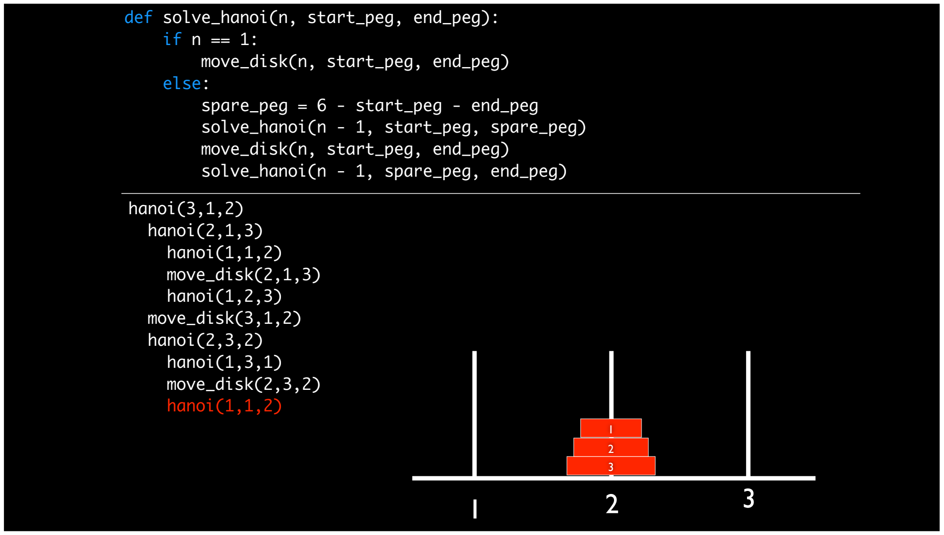
```
def solve_hanoi(n, start_peg, end_peg):
    if n == 1:
        move_disk(n, start_peg, end_peg)
    else:
        spare_peg = 6 - start_peg - end_peg
        solve_hanoi(n - 1, start_peg, spare_peg)
        move_disk(n, start_peg, end_peg)
        solve_hanoi(n - 1, spare_peg, end_peg)
```

```
hanoi(3,1,2)
  hanoi(2,1,3)
    hanoi(1,1,2)
    move_disk(2,1,3)
    hanoi(1,2,3)
  move_disk(3,1,2)
  hanoi(2,3,2)
    hanoi(1,3,1)
    move_disk(2,3,2)
    hanoi(1,1,2)
```



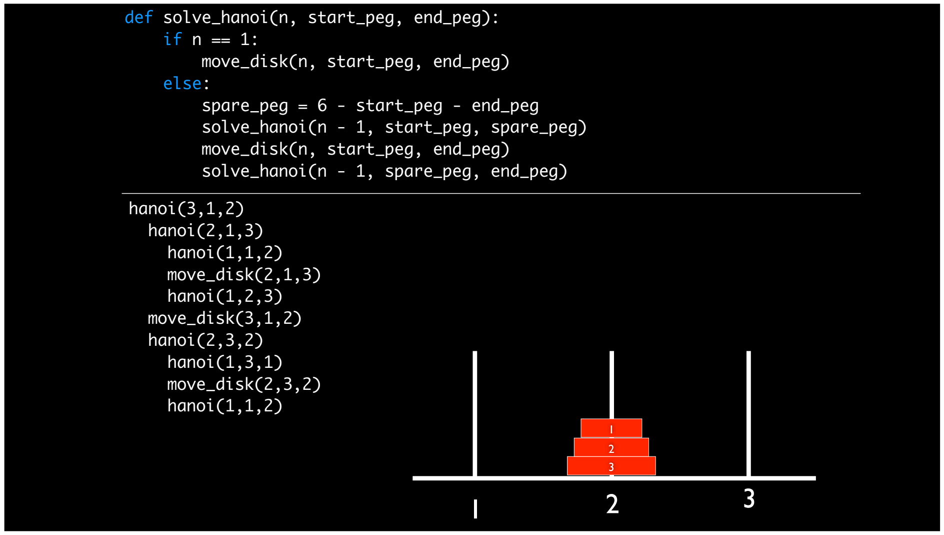
```
def solve_hanoi(n, start_peg, end_peg):
    if n == 1:
        move_disk(n, start_peg, end_peg)
    else:
        spare_peg = 6 - start_peg - end_peg
        solve_hanoi(n - 1, start_peg, spare_peg)
        move_disk(n, start_peg, end_peg)
        solve_hanoi(n - 1, spare_peg, end_peg)
```

```
hanoi(3,1,2)
  hanoi(2,1,3)
    hanoi(1,1,2)
    move_disk(2,1,3)
    hanoi(1,2,3)
  move_disk(3,1,2)
  hanoi(2,3,2)
    hanoi(1,3,1)
    move_disk(2,3,2)
    hanoi(1,1,2)
```



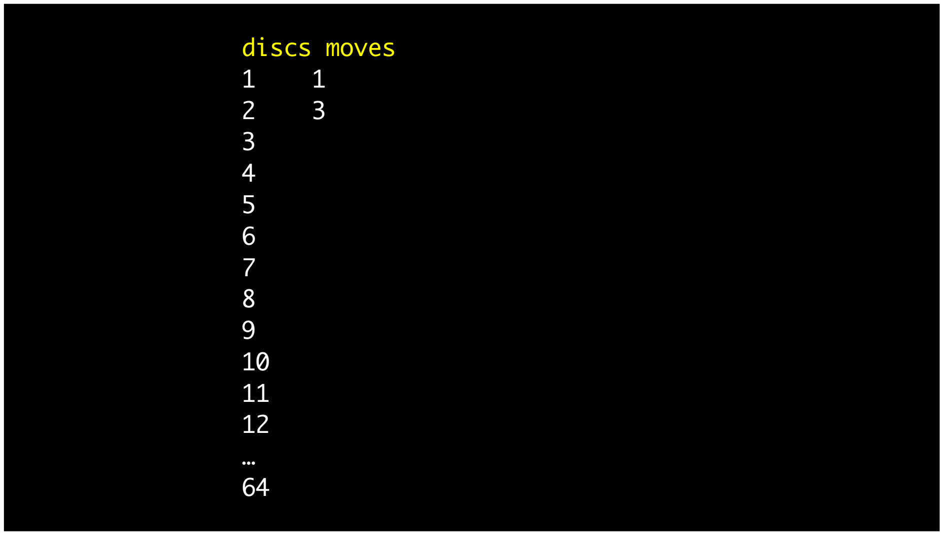
```
def solve_hanoi(n, start_peg, end_peg):
    if n == 1:
        move_disk(n, start_peg, end_peg)
    else:
        spare_peg = 6 - start_peg - end_peg
        solve_hanoi(n - 1, start_peg, spare_peg)
        move_disk(n, start_peg, end_peg)
        solve_hanoi(n - 1, spare_peg, end_peg)
```

```
hanoi(3,1,2)
  hanoi(2,1,3)
    hanoi(1,1,2)
    move_disk(2,1,3)
    hanoi(1,2,3)
  move_disk(3,1,2)
  hanoi(2,3,2)
    hanoi(1,3,1)
    move_disk(2,3,2)
    hanoi(1,1,2)
```

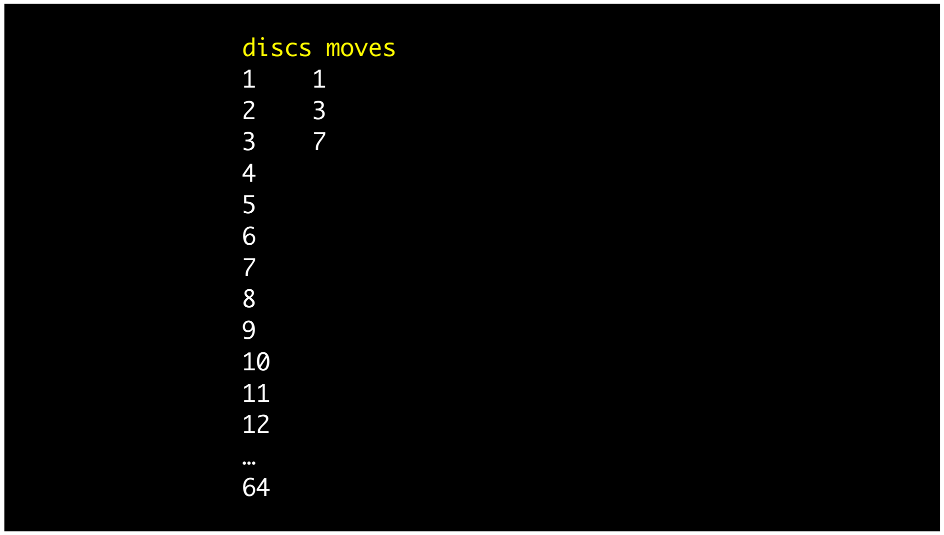


```
def solve_hanoi(n, start_peg, end_peg):
    if n == 1:
        move_disk(n, start_peg, end_peg)
    else:
        spare_peg = 6 - start_peg - end_peg
        solve_hanoi(n - 1, start_peg, spare_peg)
        move_disk(n, start_peg, end_peg)
        solve_hanoi(n - 1, spare_peg, end_peg)
```

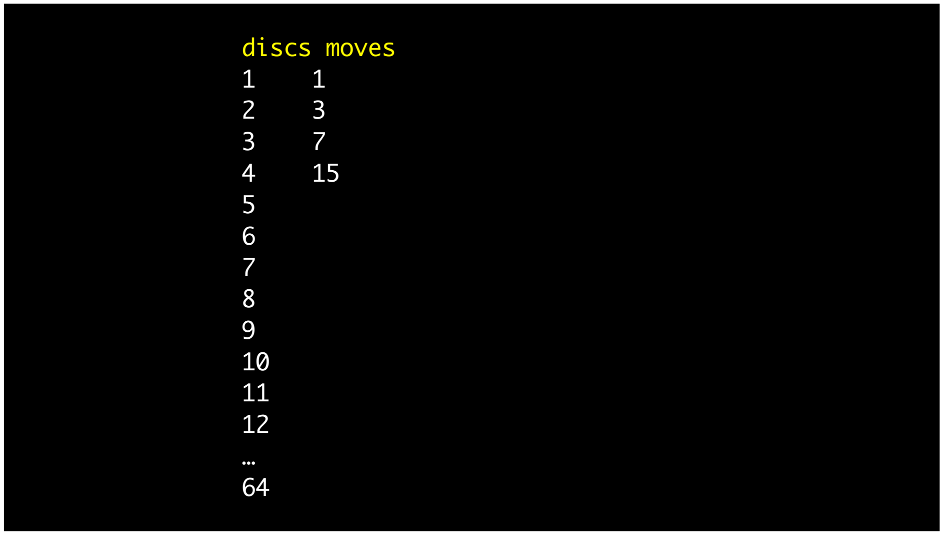
```
hanoi(3,1,2)
  hanoi(2,1,3)
    hanoi(1,1,2)
    move_disk(2,1,3)
    hanoi(1,2,3)
  move_disk(3,1,2)
  hanoi(2,3,2)
    hanoi(1,3,1)
    move_disk(2,3,2)
    hanoi(1,1,2)
```



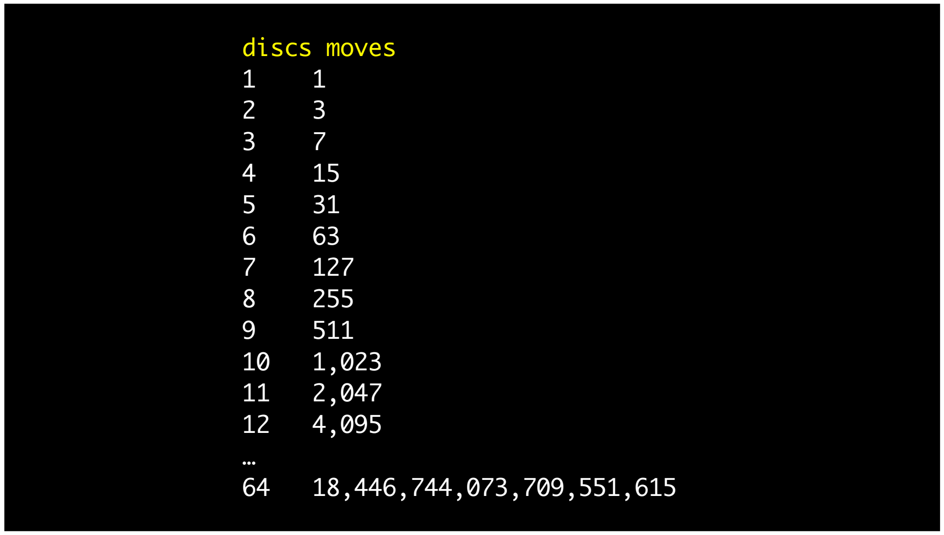
```
discs moves
1    1
2    3
3
4
5
6
7
8
9
10
11
12
…
64
```



```
discs moves
1    1
2    3
3    7
4
5
6
7
8
9
10
11
12
…
64
```



```
discs moves
1    1
2    3
3    7
4    15
5
6
7
8
9
10
11
12
…
64
```



```
discs moves
1    1
2    3
3    7
4    15
5    31
6    63
7    127
8    255
9    511
10   1,023
11   2,047
12   4,095
…
64   18,446,744,073,709,551,615
```