Functional Abstraction

Announcements

Office Hours: You Should Go!

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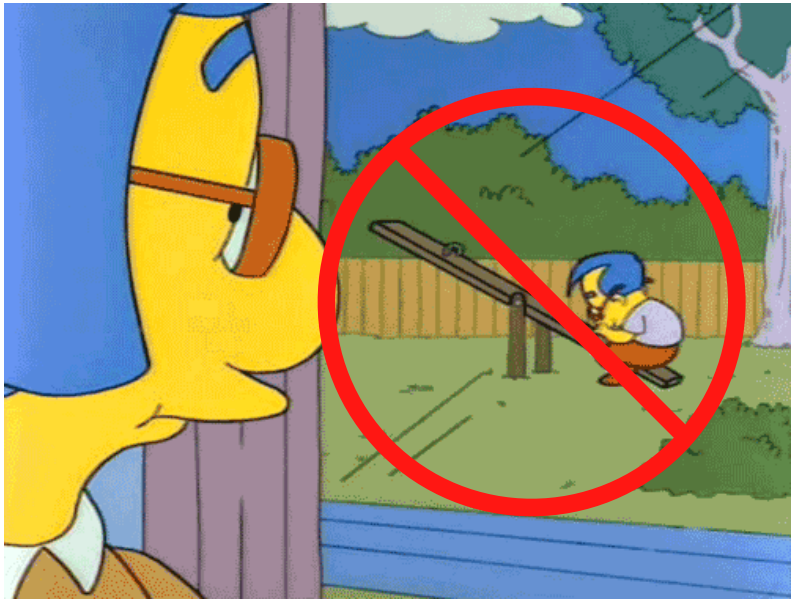
You are not alone!



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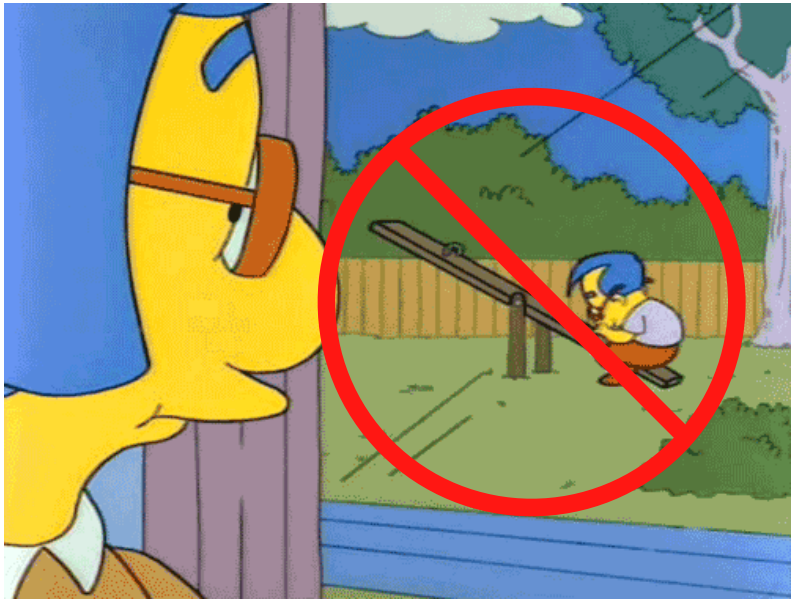

https://cs61a.org/office-hours/

Partial Function Application & Currying

Returning a Function to Wait for More Arguments

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def make_adder(n):
    def adder(k):
        return n + k
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add(3, 4) applies addition to the arguments 3 and 4, while
make_adder(3) *partially applies* addition, but is still waiting for k.

Function Currying

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(Demo)

Lambda Function Environments

Environment Diagrams with Lambda

A lambda function's parent is the current frame in which the lambda expression is evaluated

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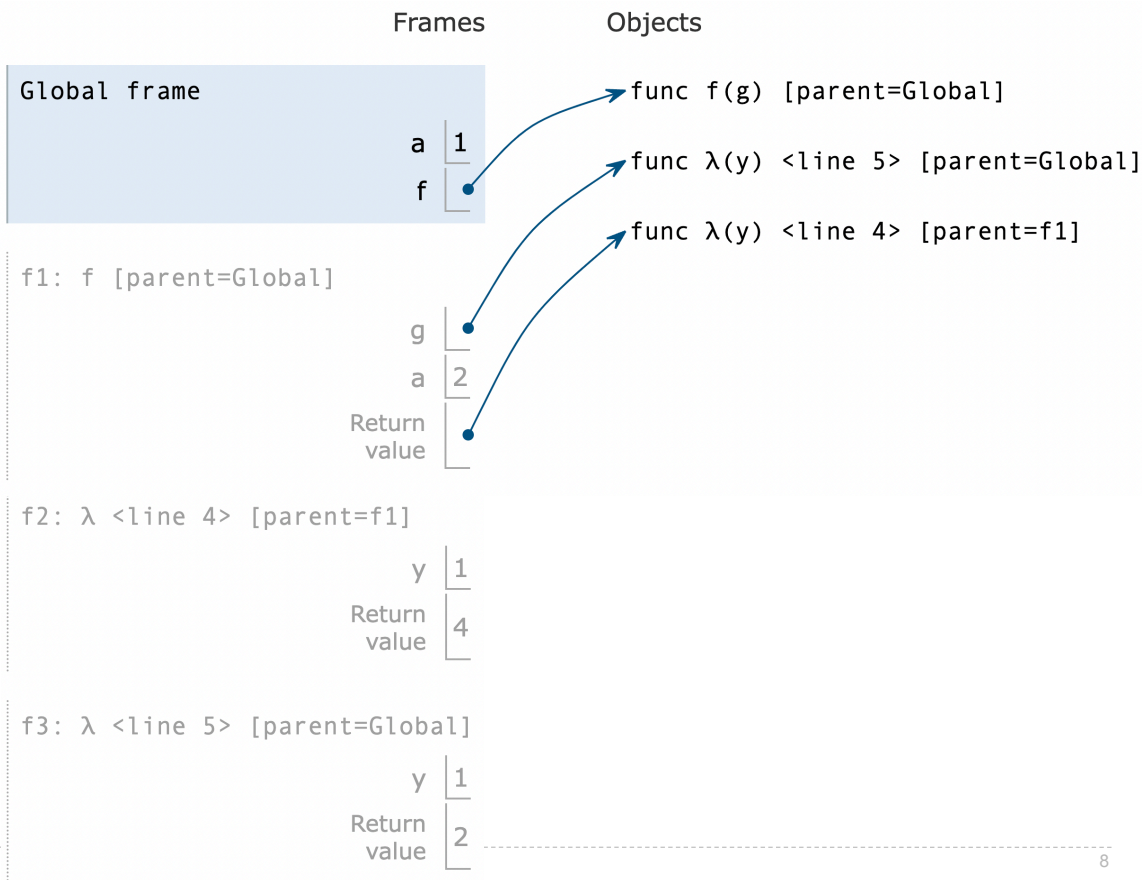

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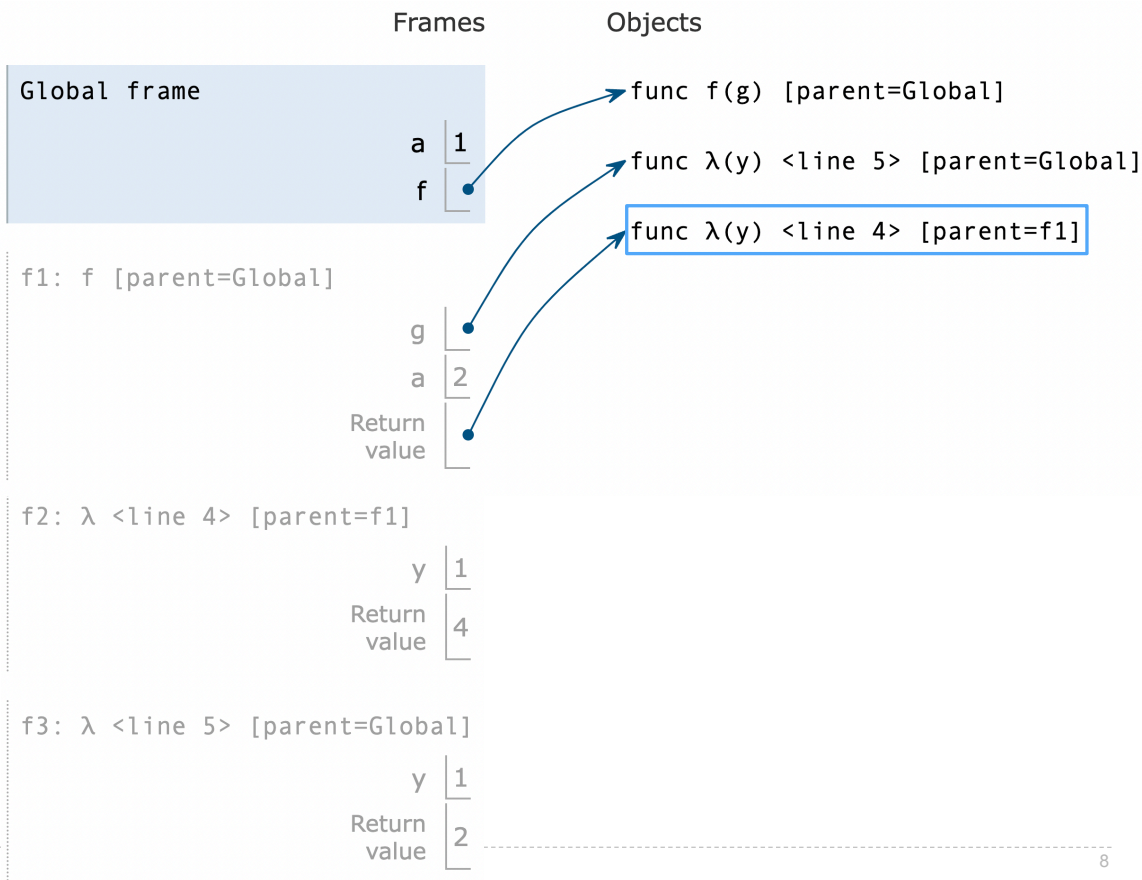

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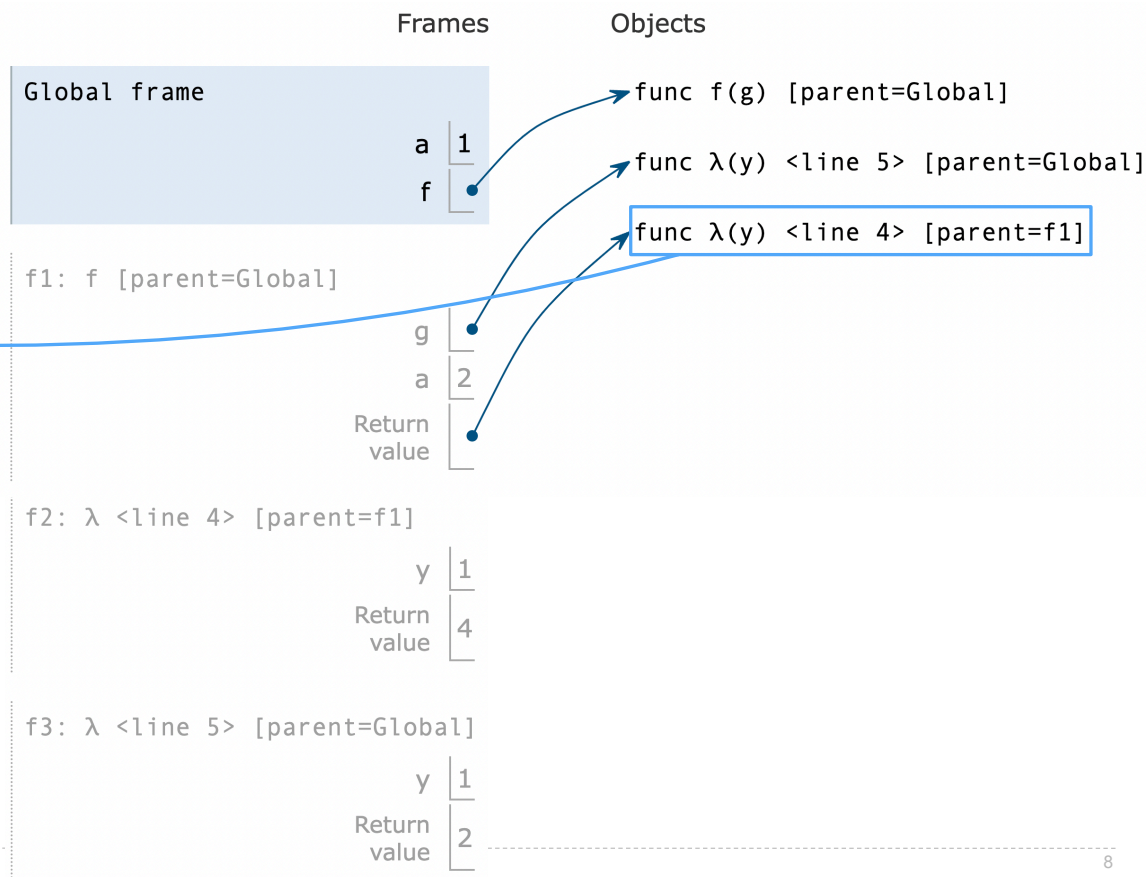

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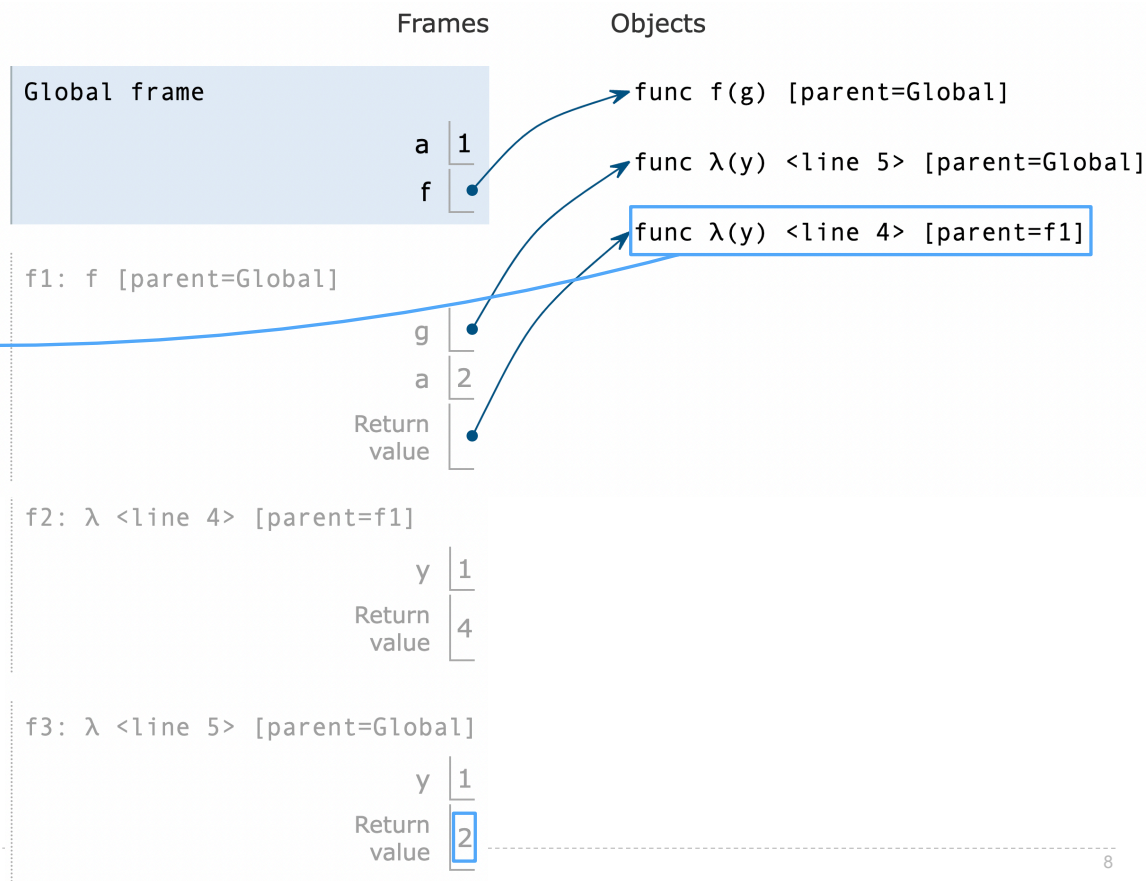

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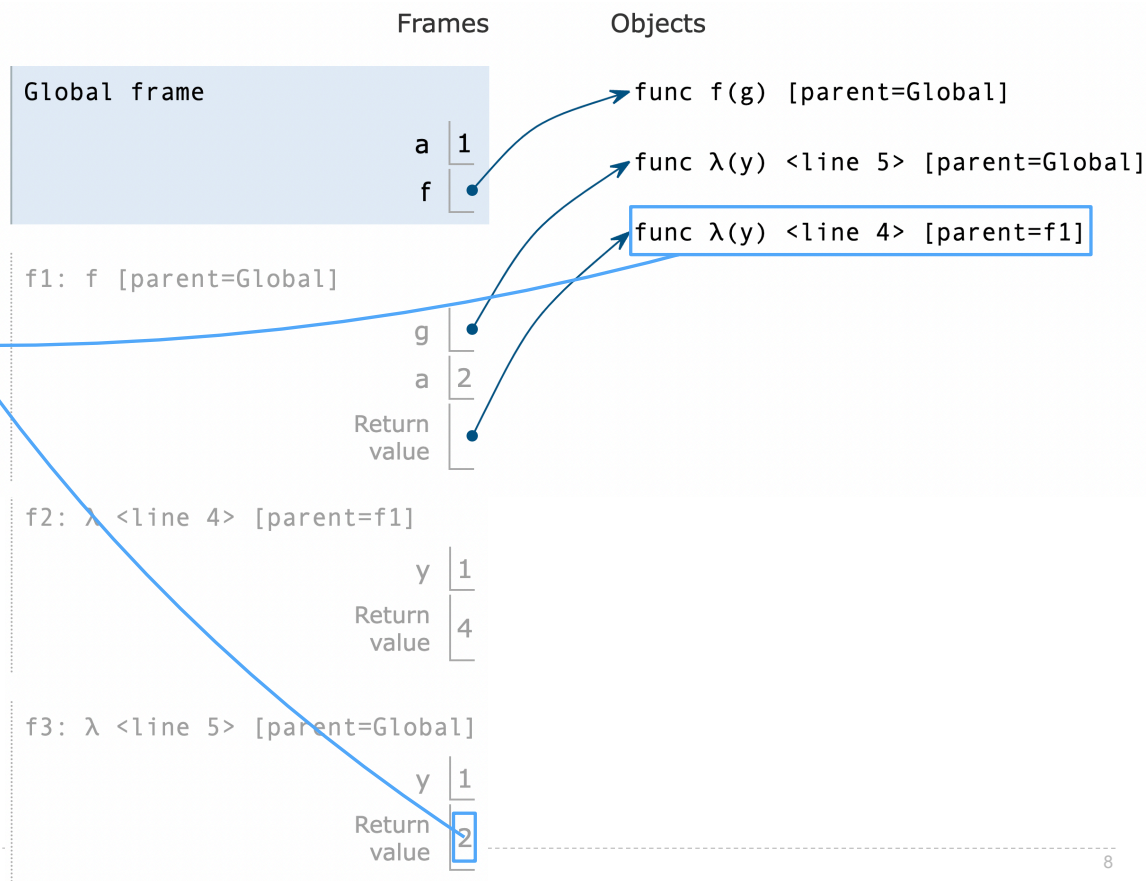

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Decorators

Function Decorators

(Demo)

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def triple(x):
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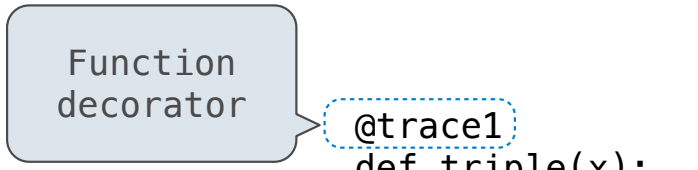


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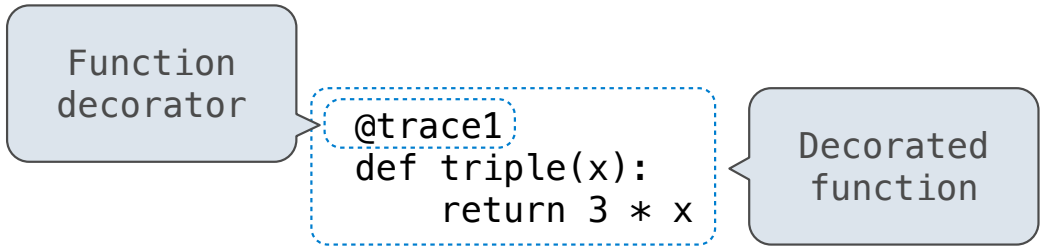

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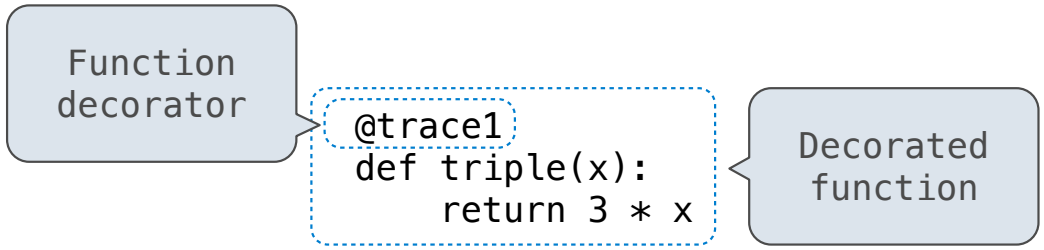

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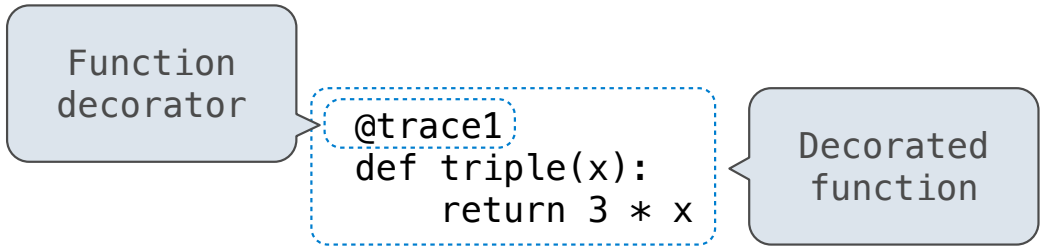

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def triple(x):
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triple = trace1(triple)
```

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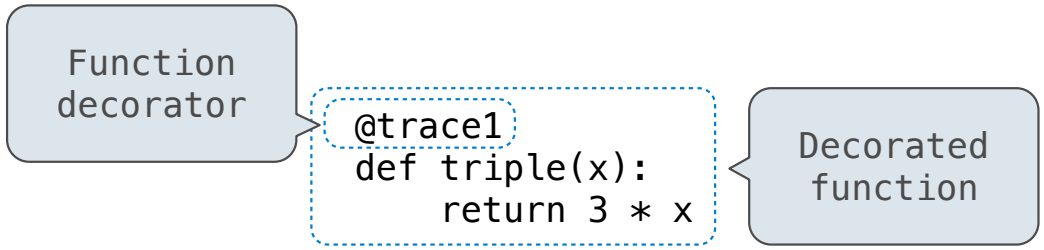

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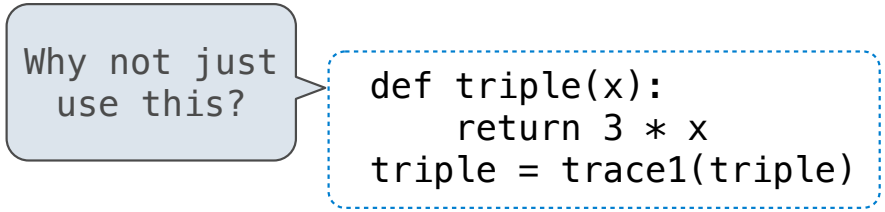

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Designing Functions

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Abstraction

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If the name "square" were bound to a built-in function, sum_squares would still have the same behavior.

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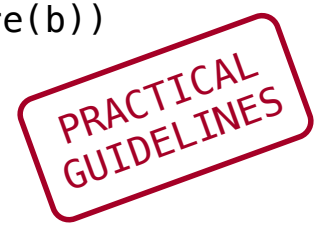

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Errors & Tracebacks

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