Higher-Order Environments

Announcements

Environments for Higher-Order Functions

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(Demo)

Names can be Bound to Functional Arguments

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3
4  def square(x):
5      return x * x
6
7  result = apply_twice(square, 2)
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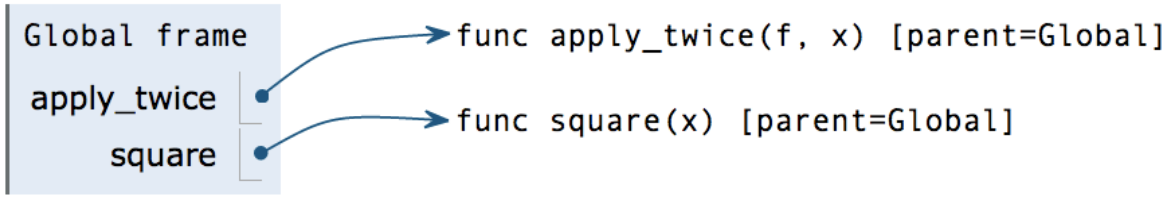

pythontutor.com/composingprograms.html#code=def%20apply_twice%28f,%20x%29%3A%0A%20%20%20%20return%20f%28f%28x%29%29%0A%20%20%20%20%0Adef%20square%28x%29%3A%0A%20%20%20%20return%20x%20*%20x%0A%20%20%20%20%0Aresult%20%3D%20apply_twice%28square,%202%29&mode=display&origin=composingprograms.js&cumulative=true&py=3&rawInputLstJSON=[]&curInstr=0

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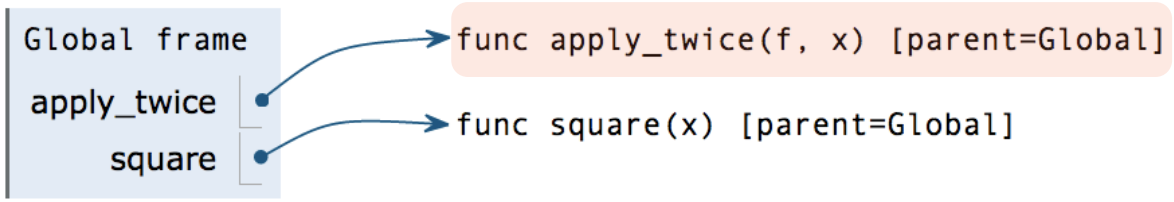

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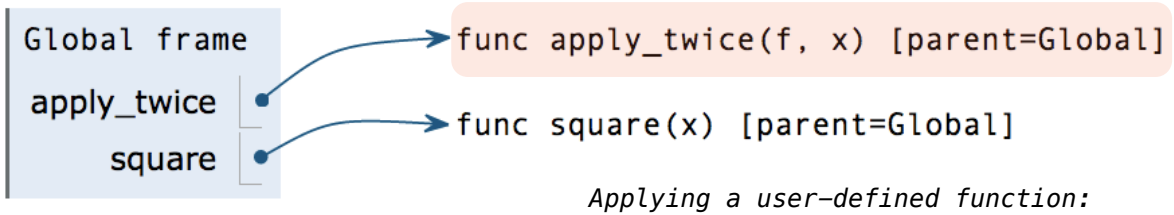

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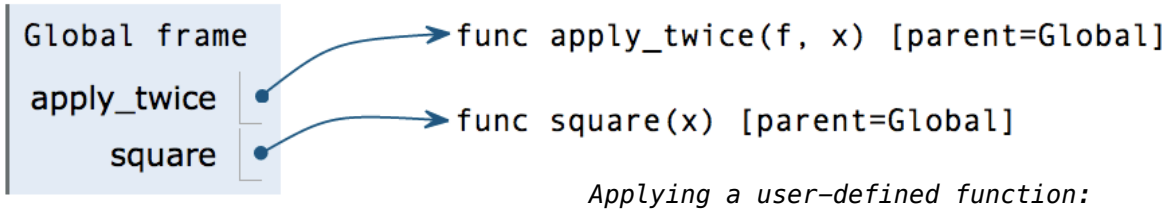

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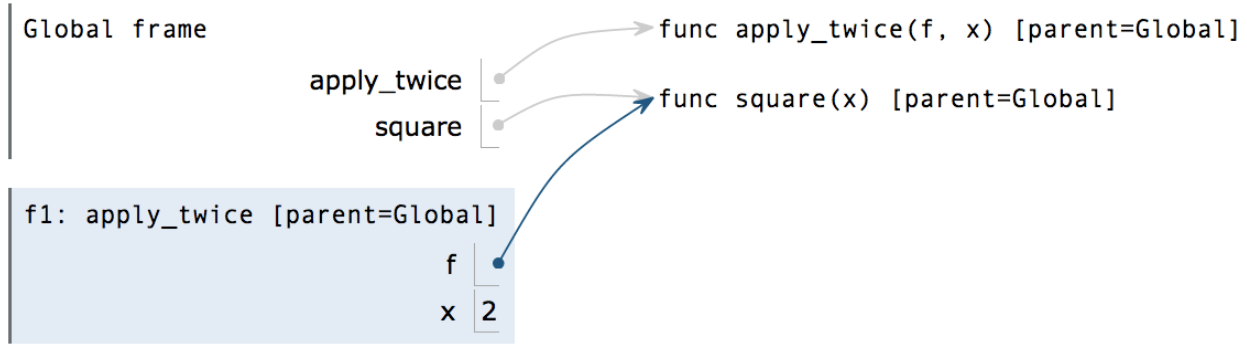

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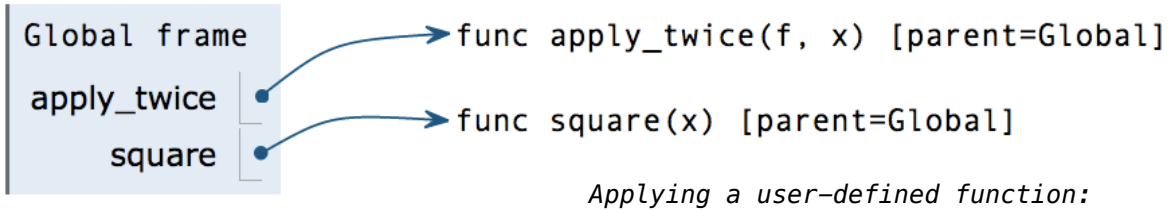

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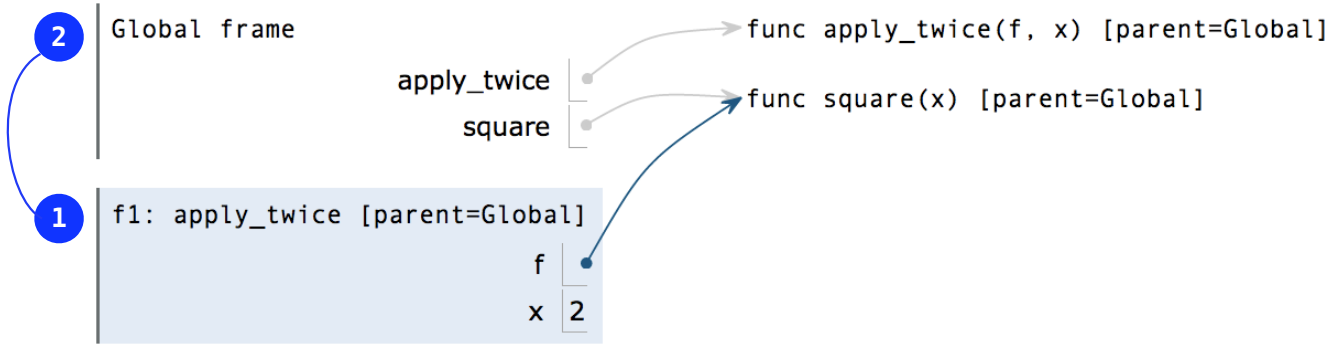

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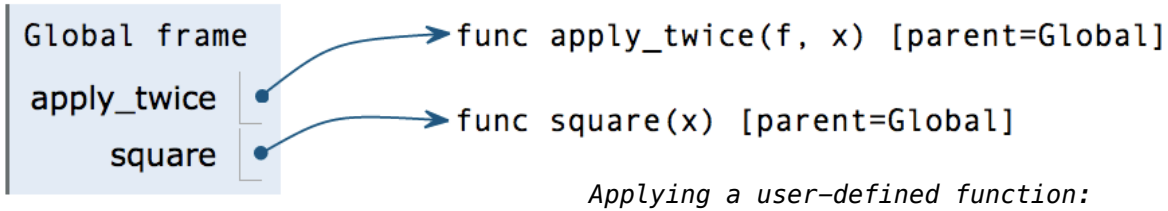

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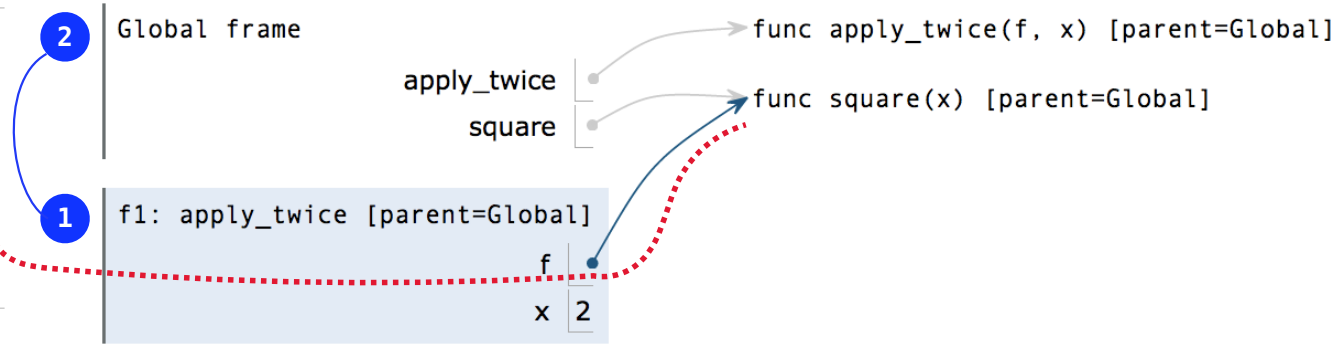

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Types of Higher-Order Functions

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(Demo)

Functions as Return Values

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```
def make_adder(n):
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    >>> add_three = make_adder(3)
    >>> add_three(4)
    7
    """
    def adder(k):
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    return adder
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A function that returns a function

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A def statement within another def statement

Can refer to names in the enclosing function

Call Expressions as Operator Expressions

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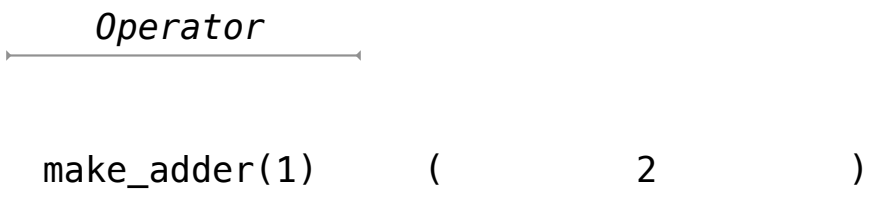
```
make_adder(1)     (         2         )
```

Call Expressions as Operator Expressions



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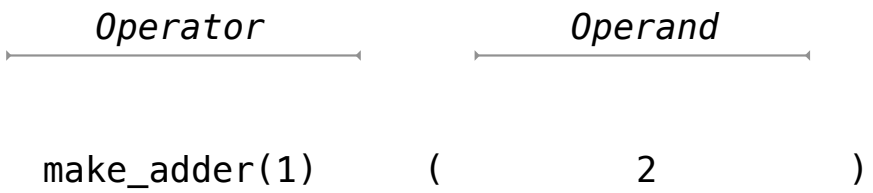


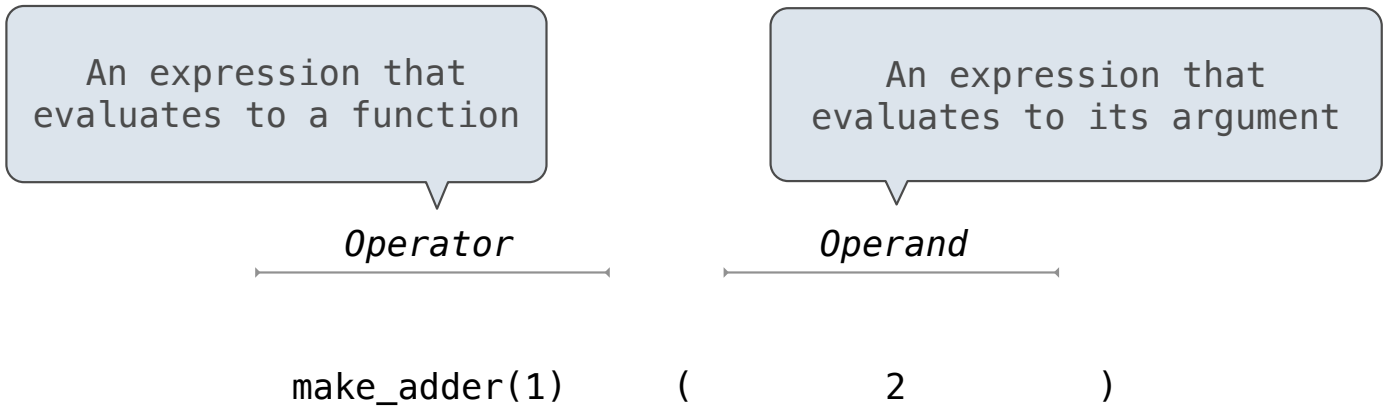
Call Expressions as Operator Expressions

An expression that evaluates to a function

Operator *Operand*

```
make_adder(1)     (         2         )
```

Call Expressions as Operator Expressions



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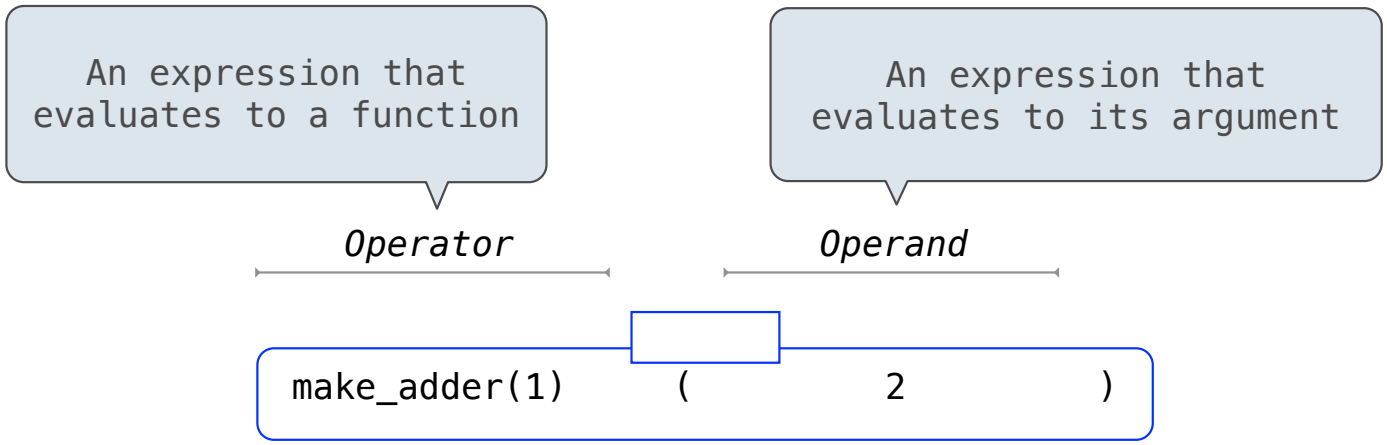

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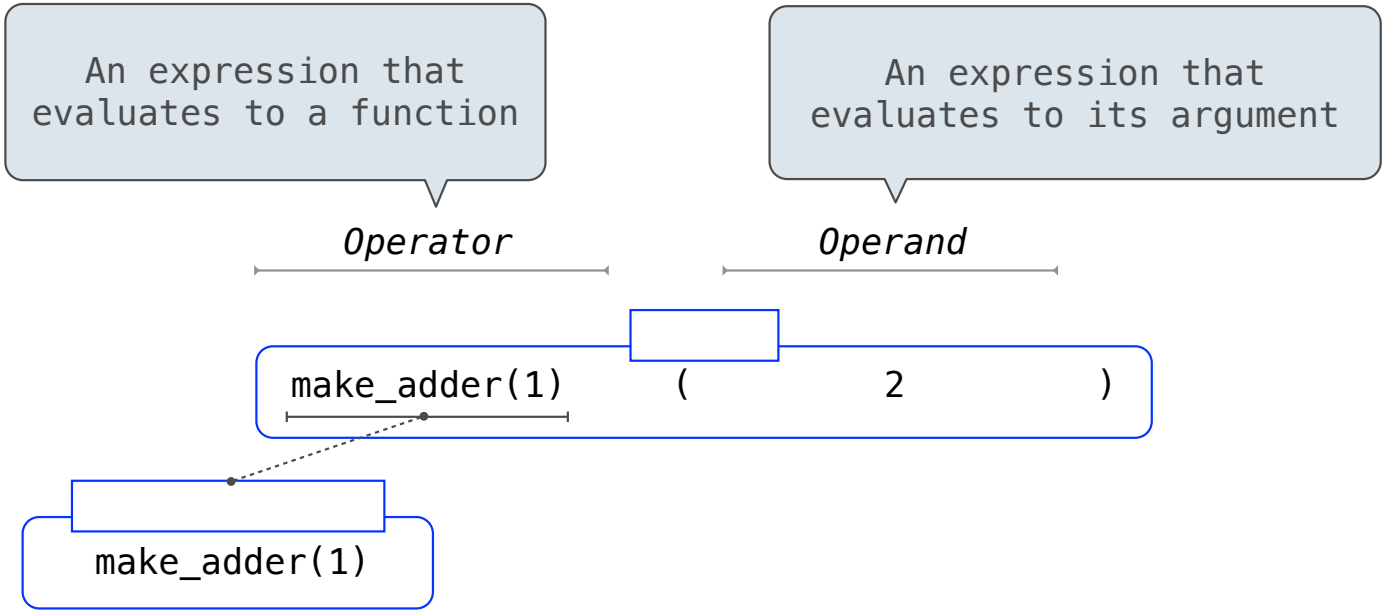

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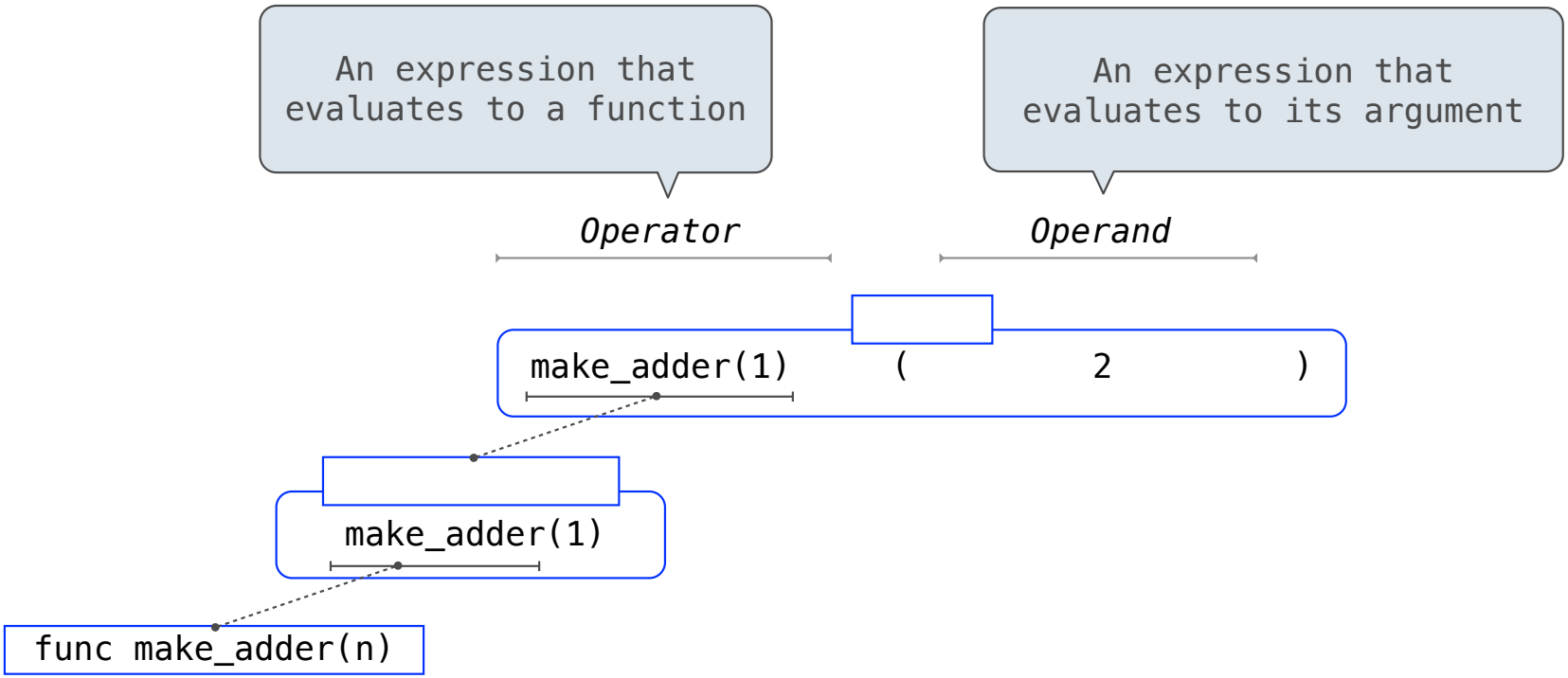

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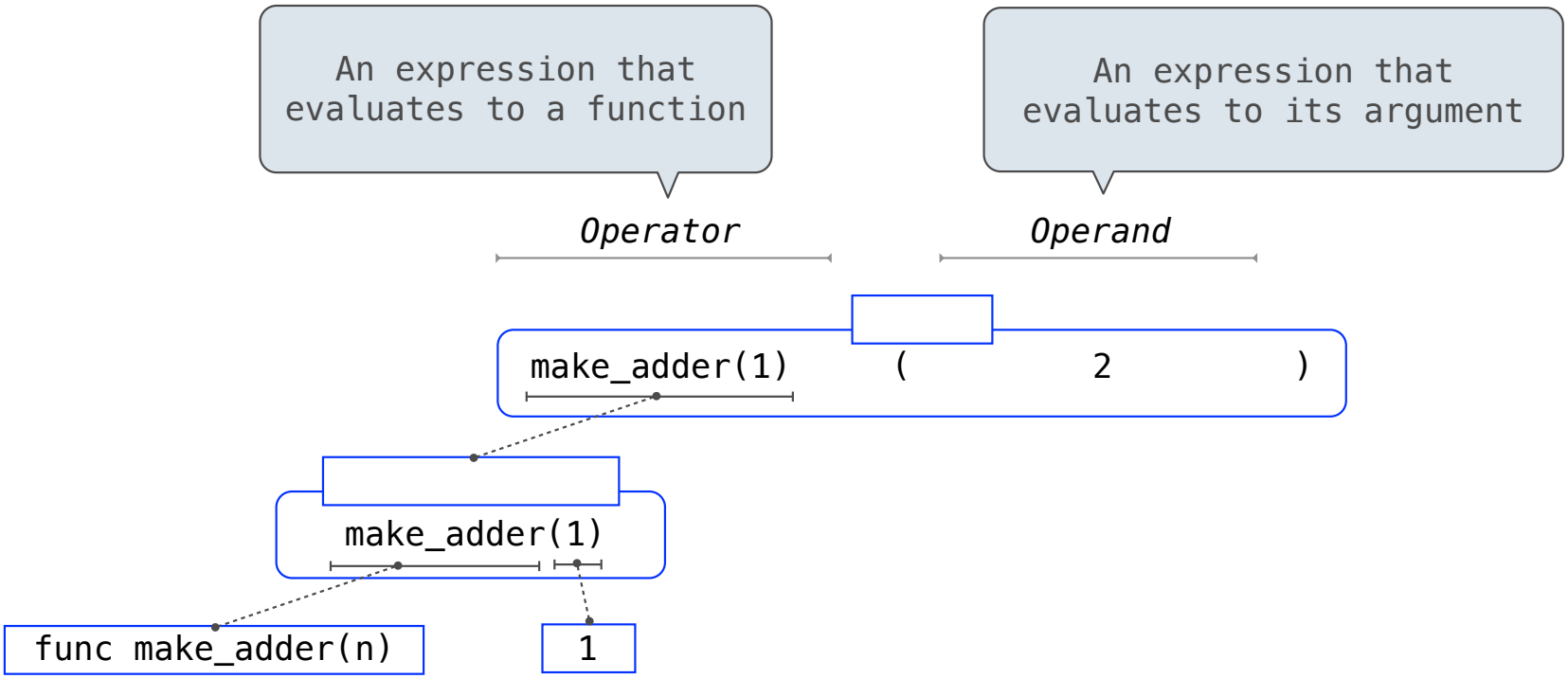

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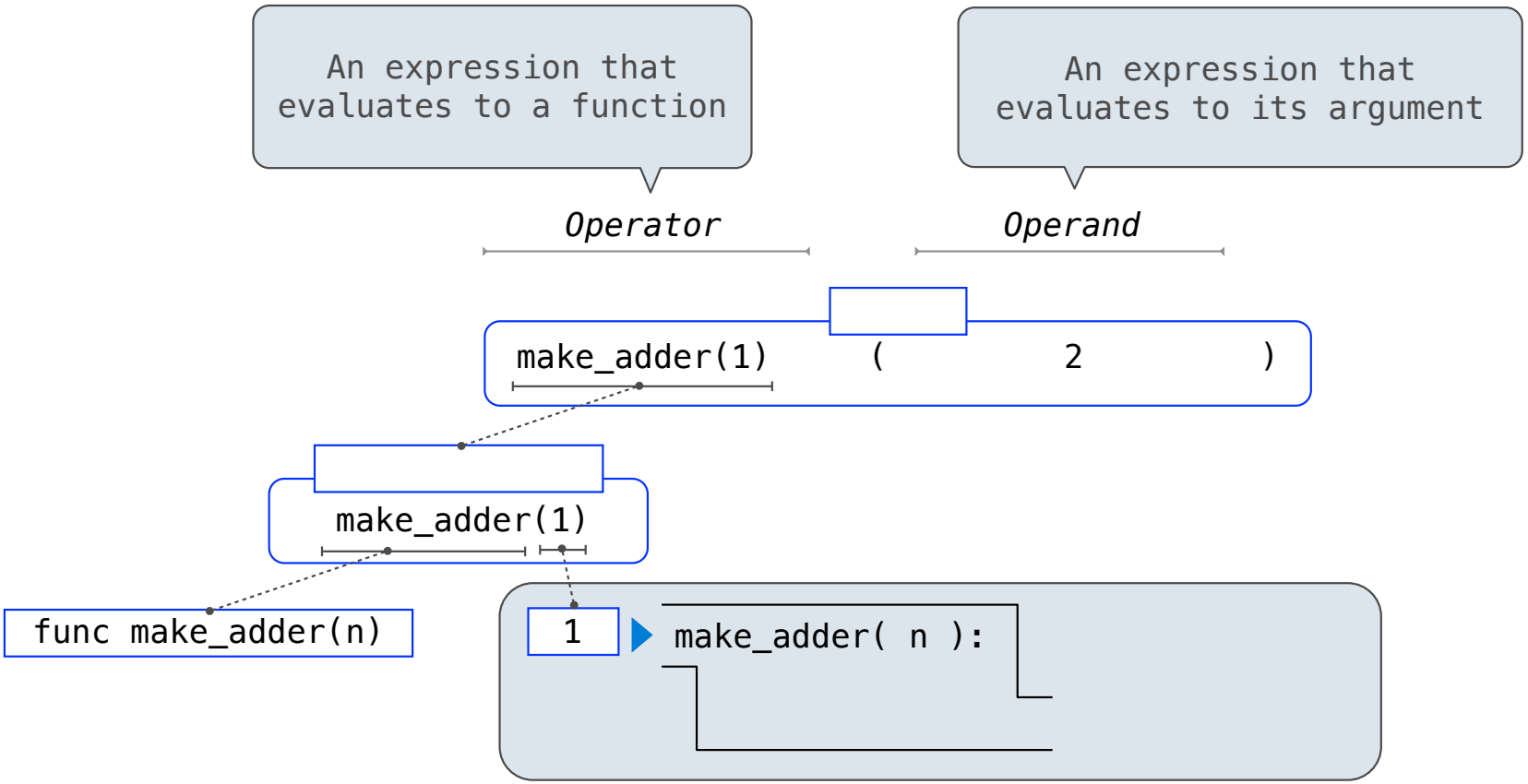

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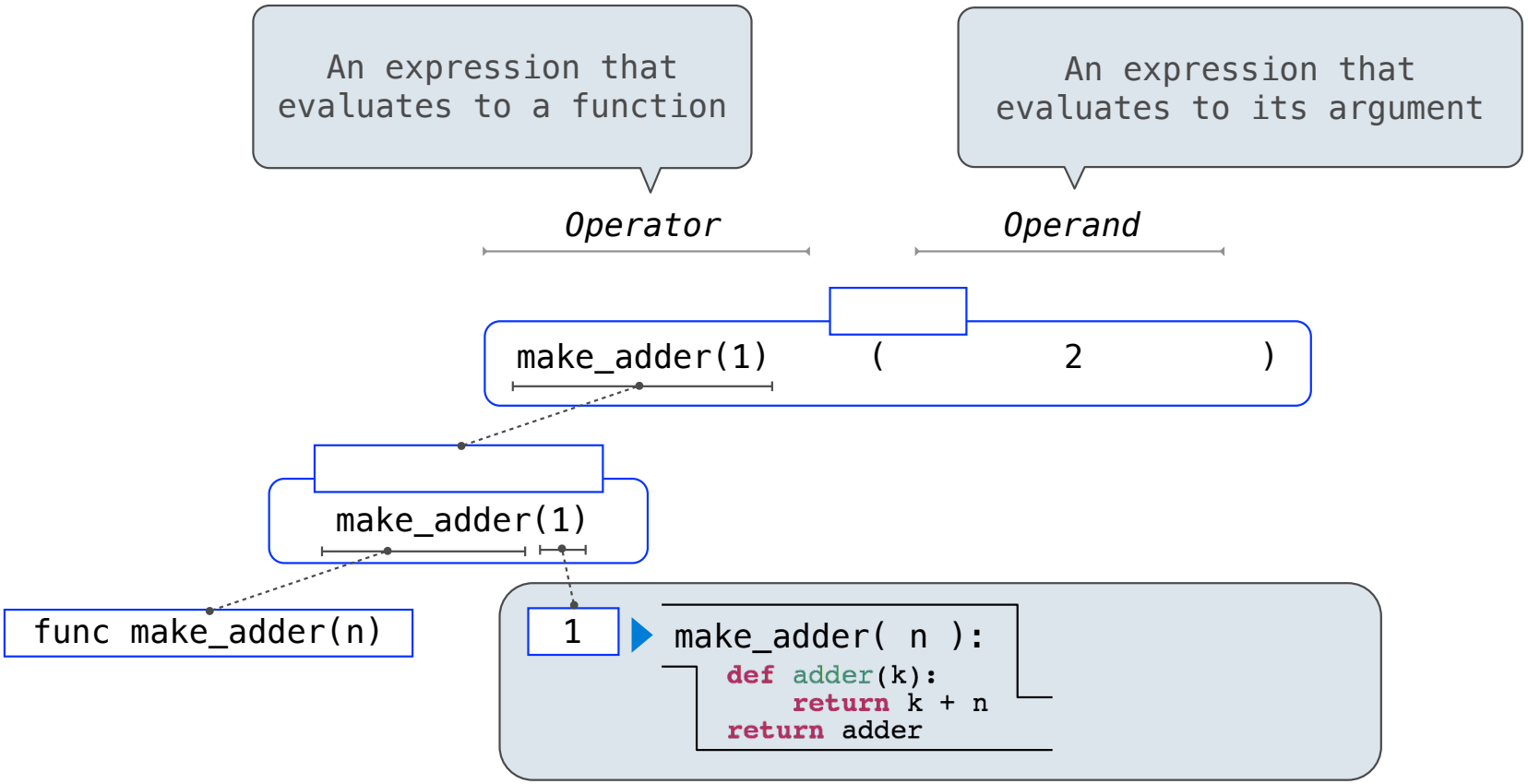

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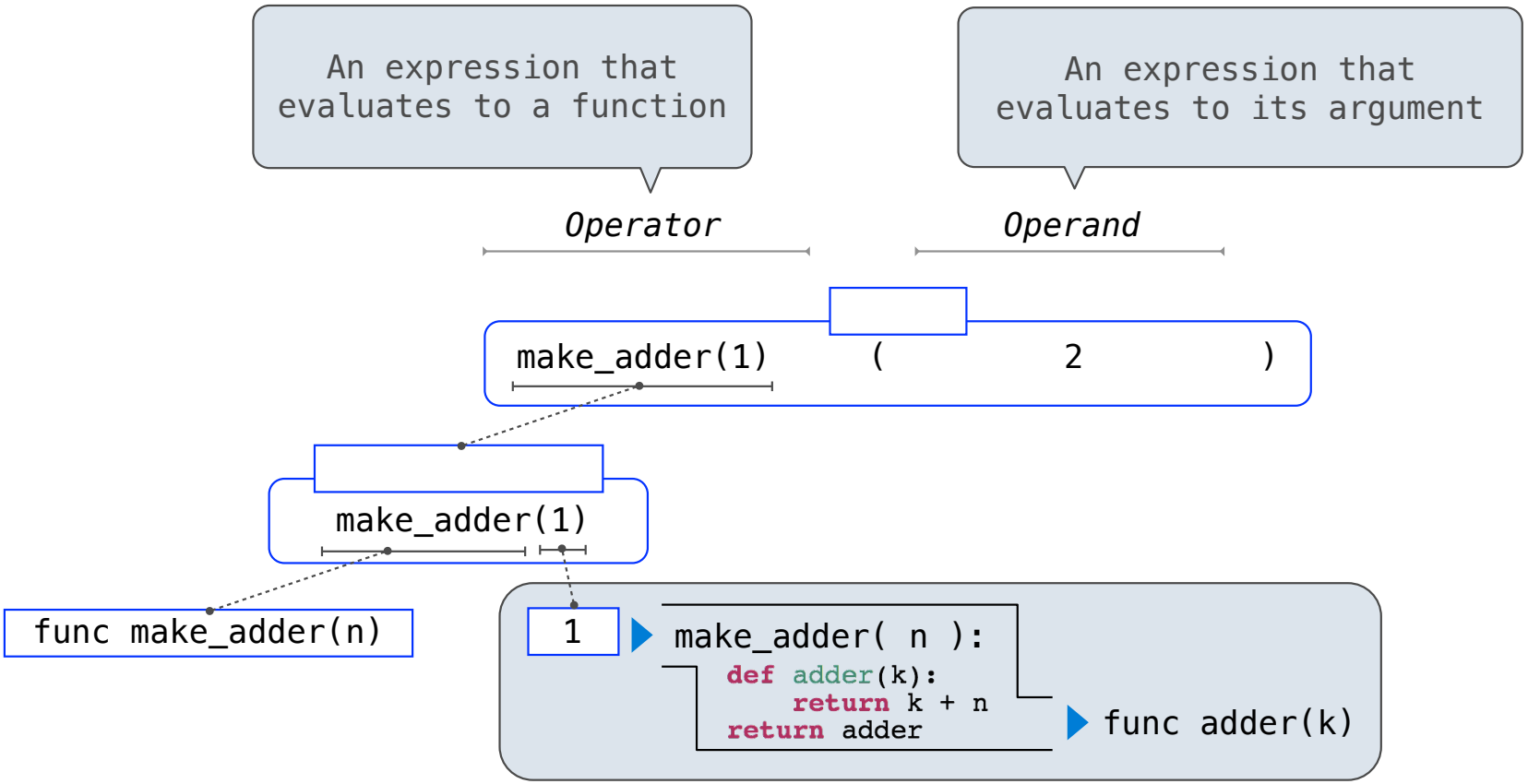

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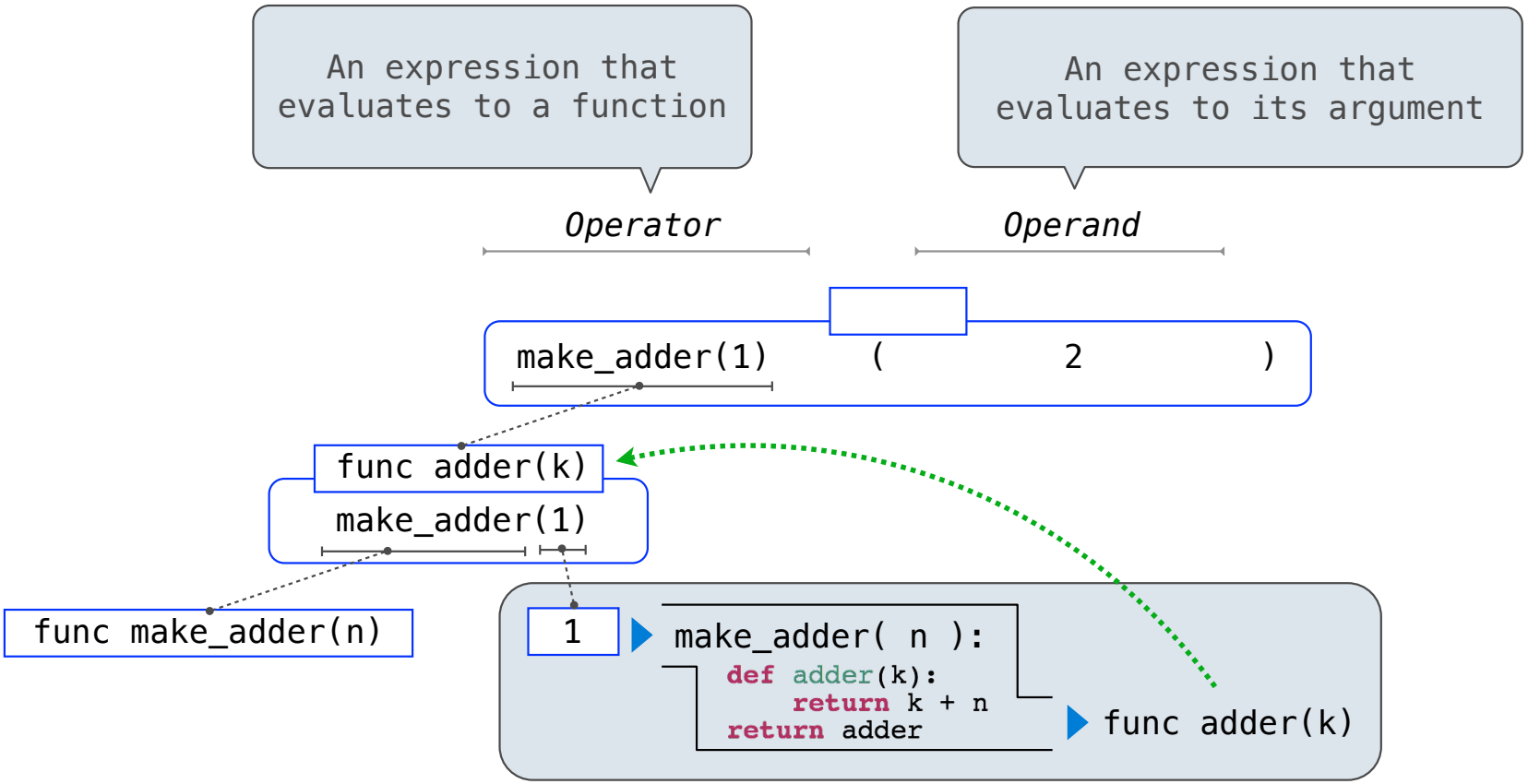

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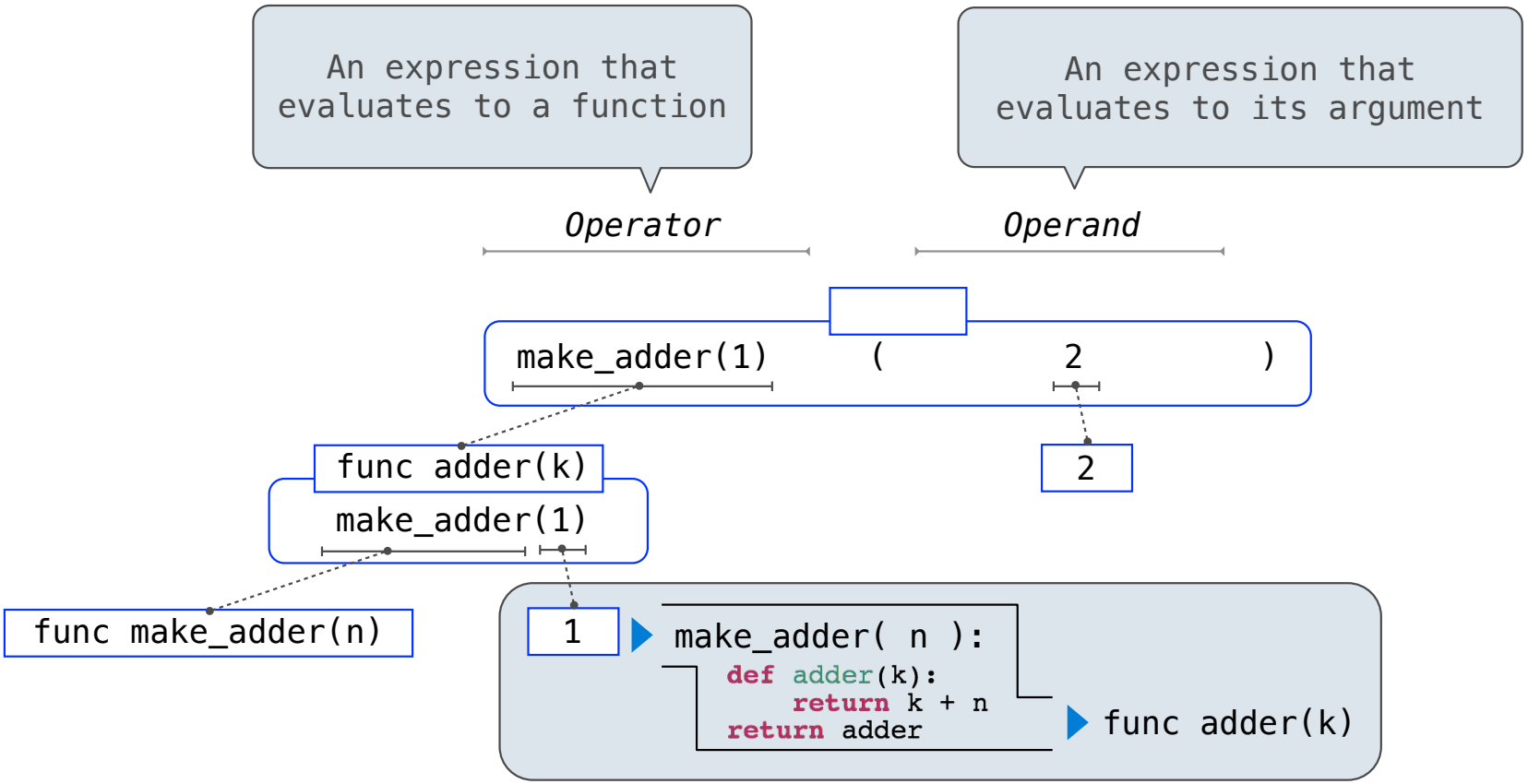

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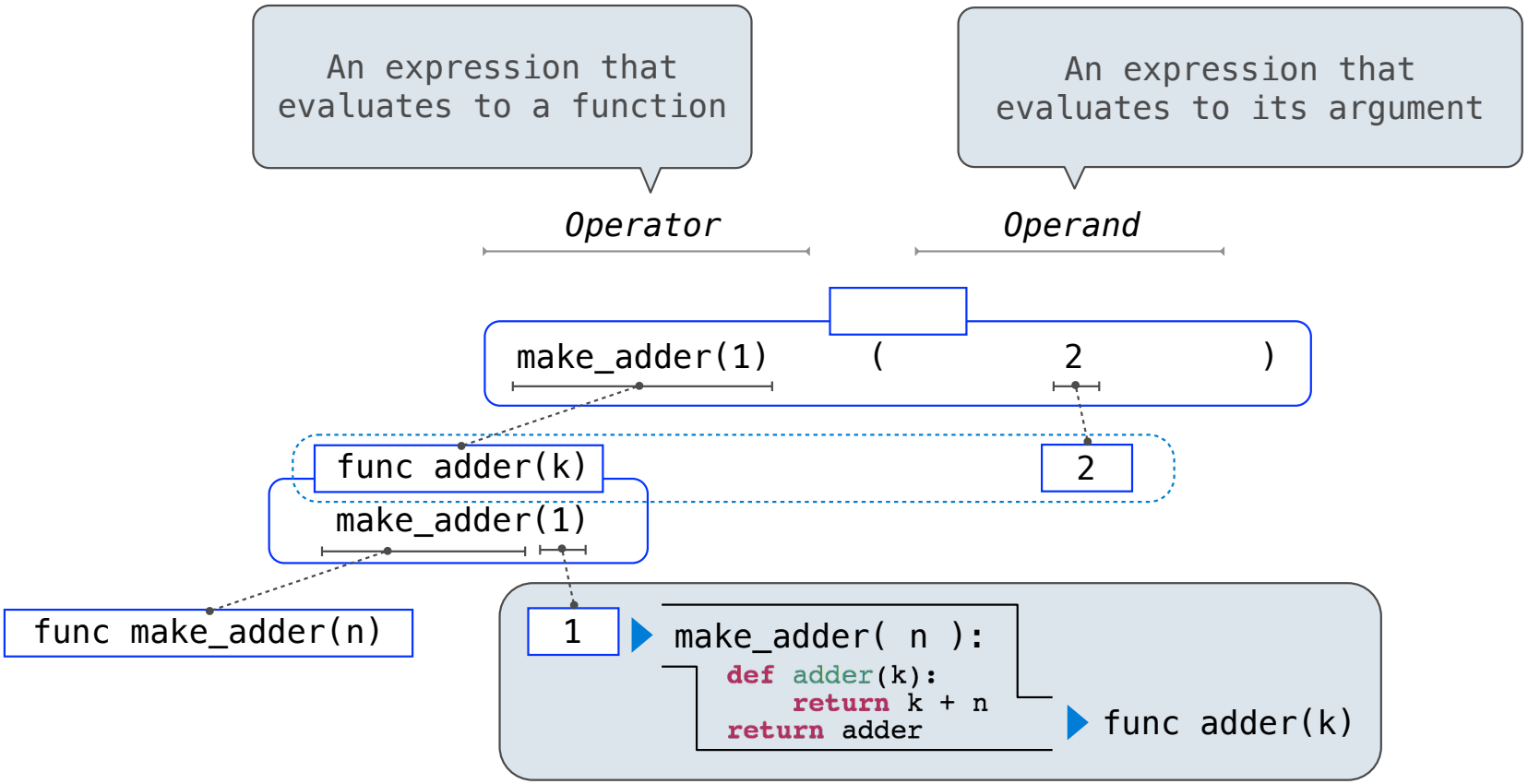

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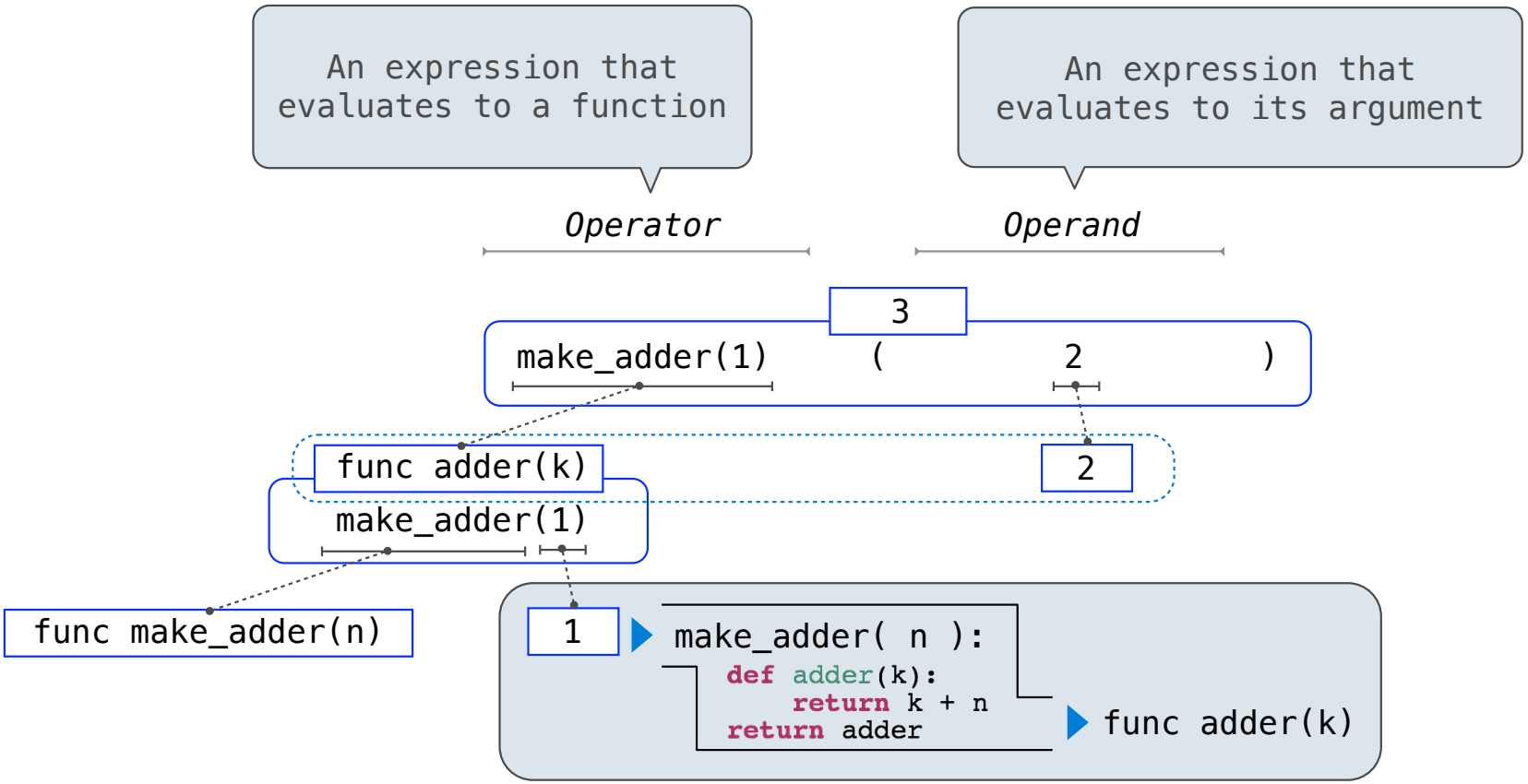

Environments for Nested Definitions

(Demo)

Environment Diagrams for Nested Def Statements

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1  def make_adder(n):
2      def adder(k):
3          return k + n
4      return adder
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6  add_three = make_adder(3)
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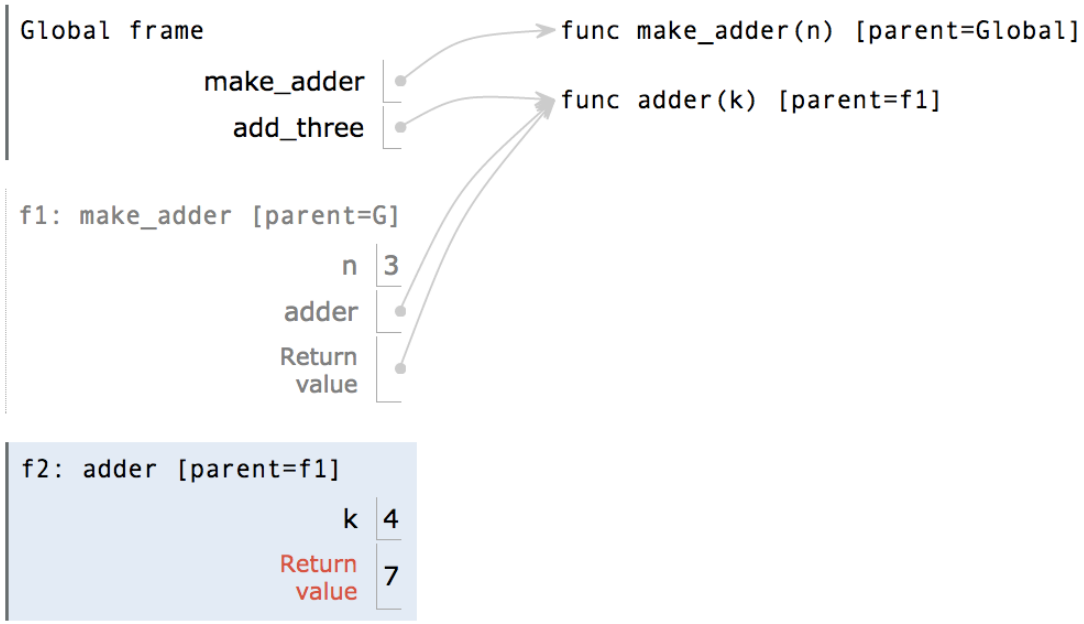

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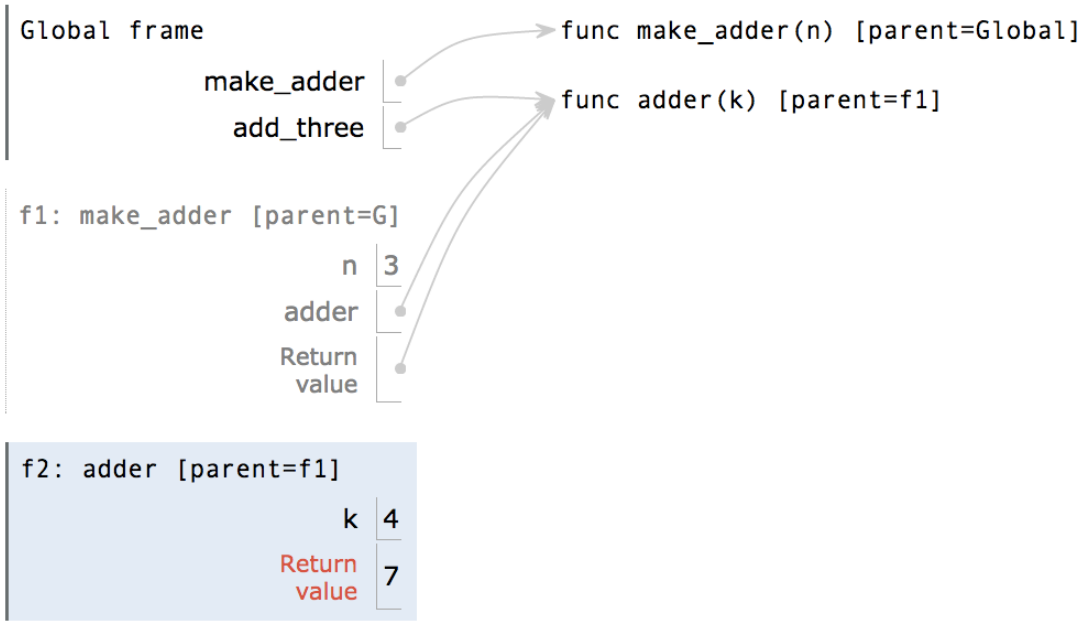

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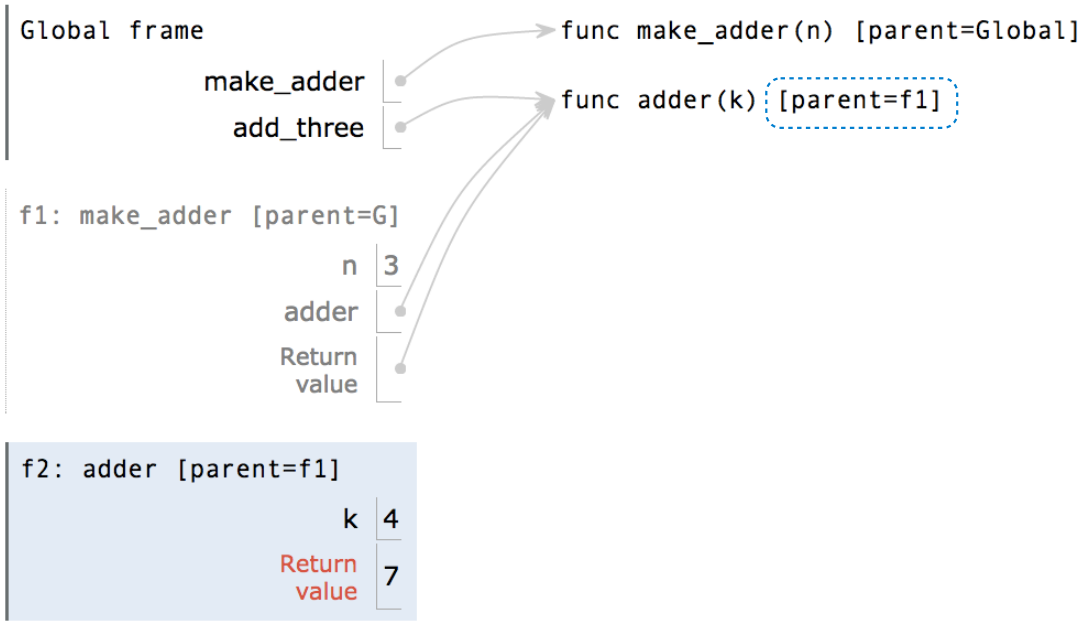

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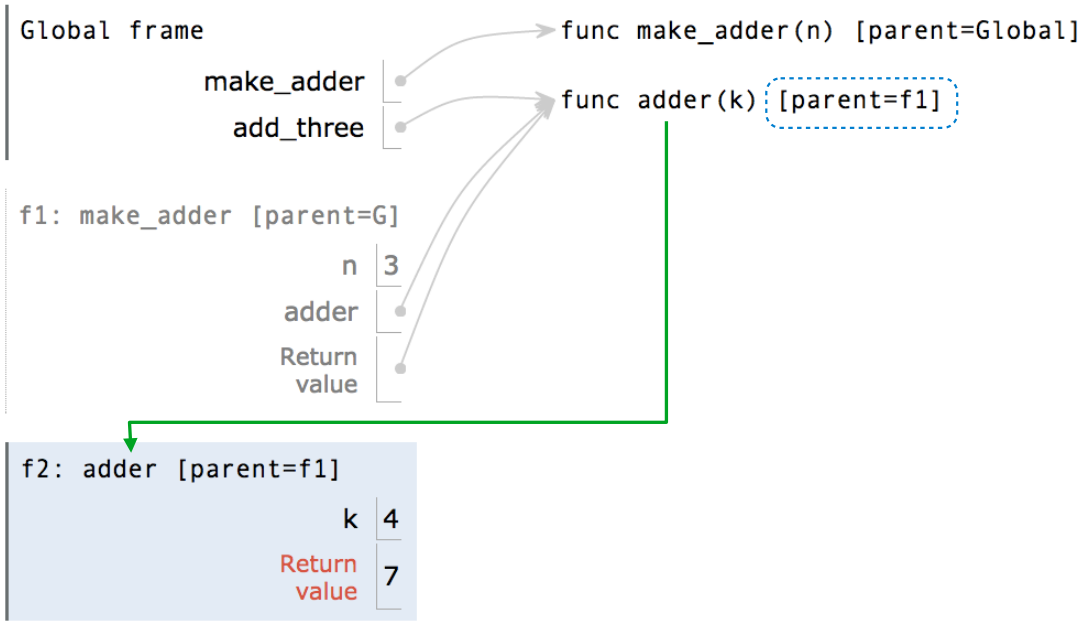

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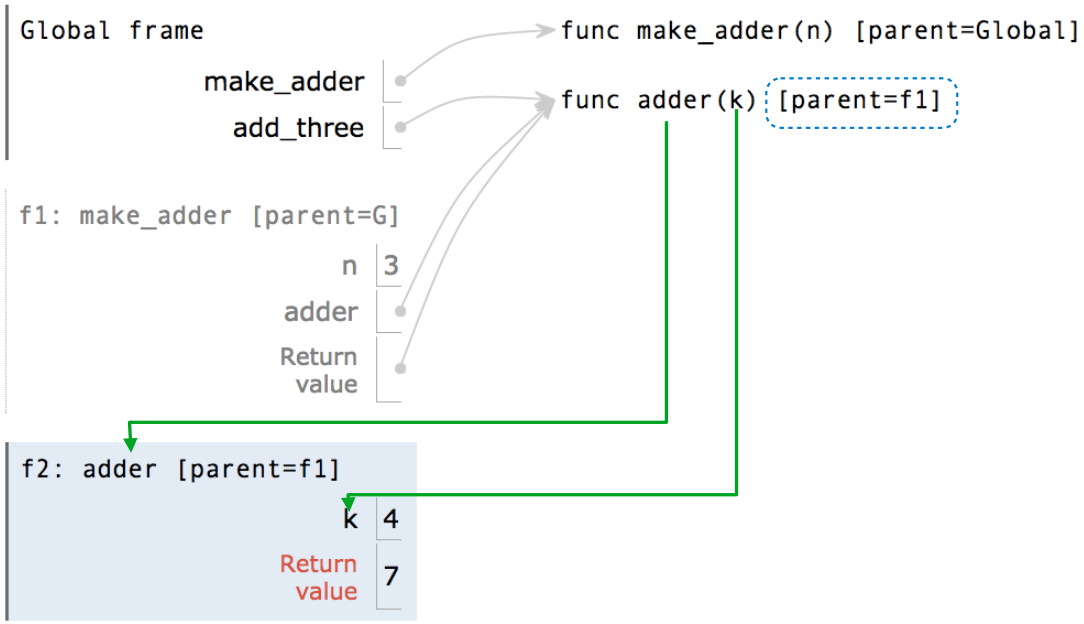

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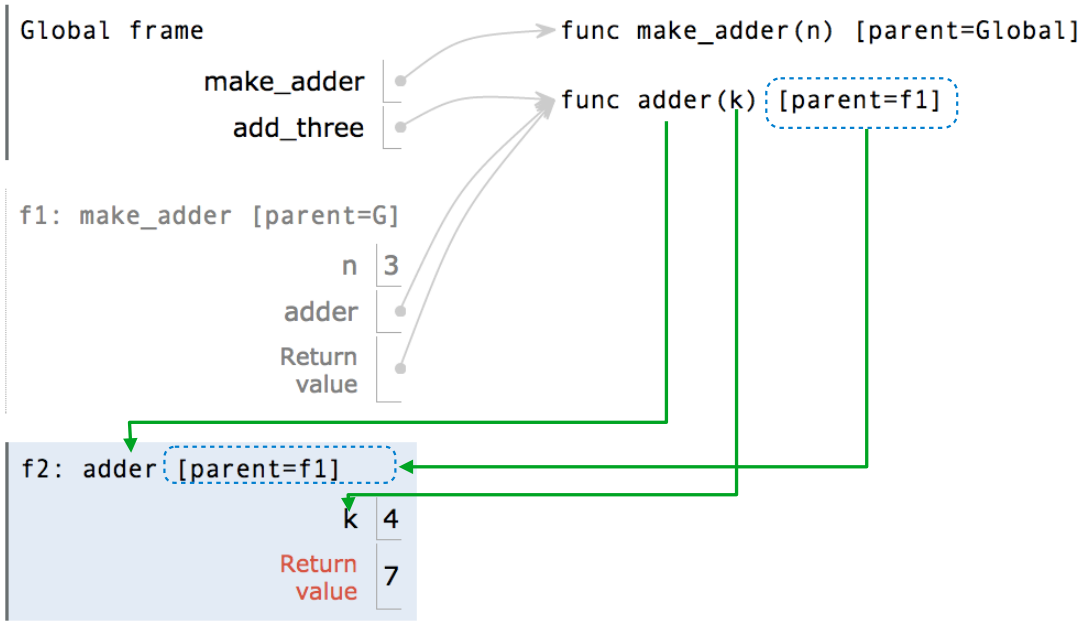

http://pythontutor.com/composingprograms.html#code=def%20make_adder%28n%29%3A%0A%20%20%20%20def%20adder%28k%29%3A%0A%20%20%20%20%20%20%20%20return%20k%20%2B%20n%0A%20%20%20%20return%20adder%0A%20%20%20%20%0Athree_more_than%20%3D%20make_adder%283%29%0Aresult%20%3D%20three_more_than%284%29&cumulative=false&curInstr=0&mode=display&origin=composingprograms.js&py=3&rawInputLstJSON=%5B%5D

Environment Diagrams for Nested Def Statements

Nested def

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1  def make_adder(n):
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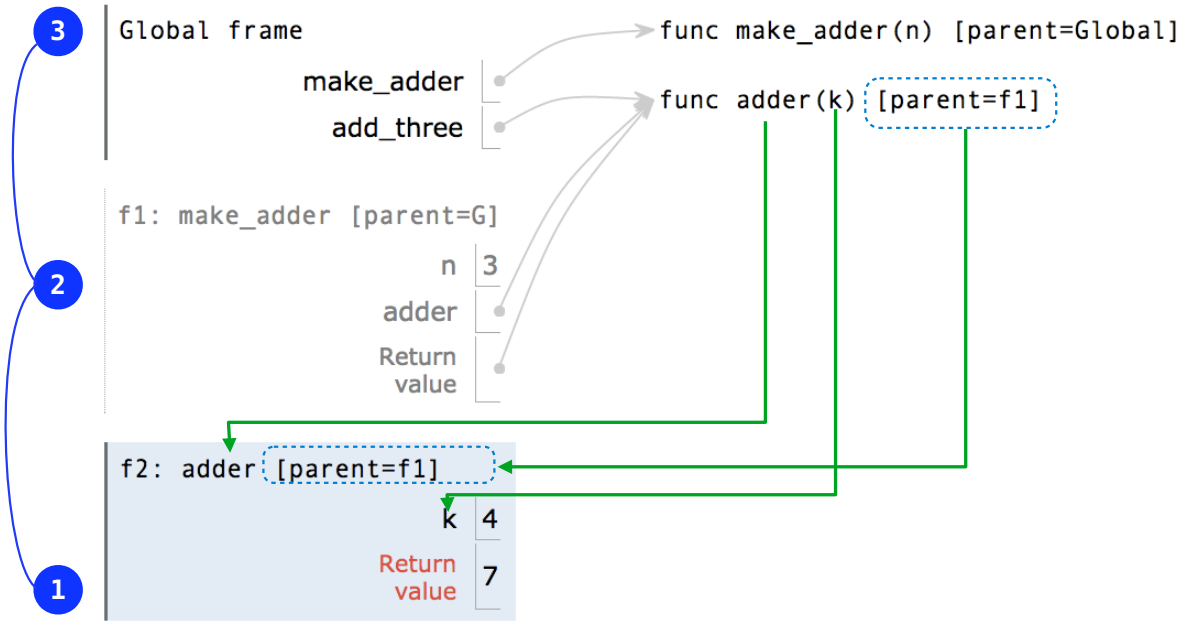

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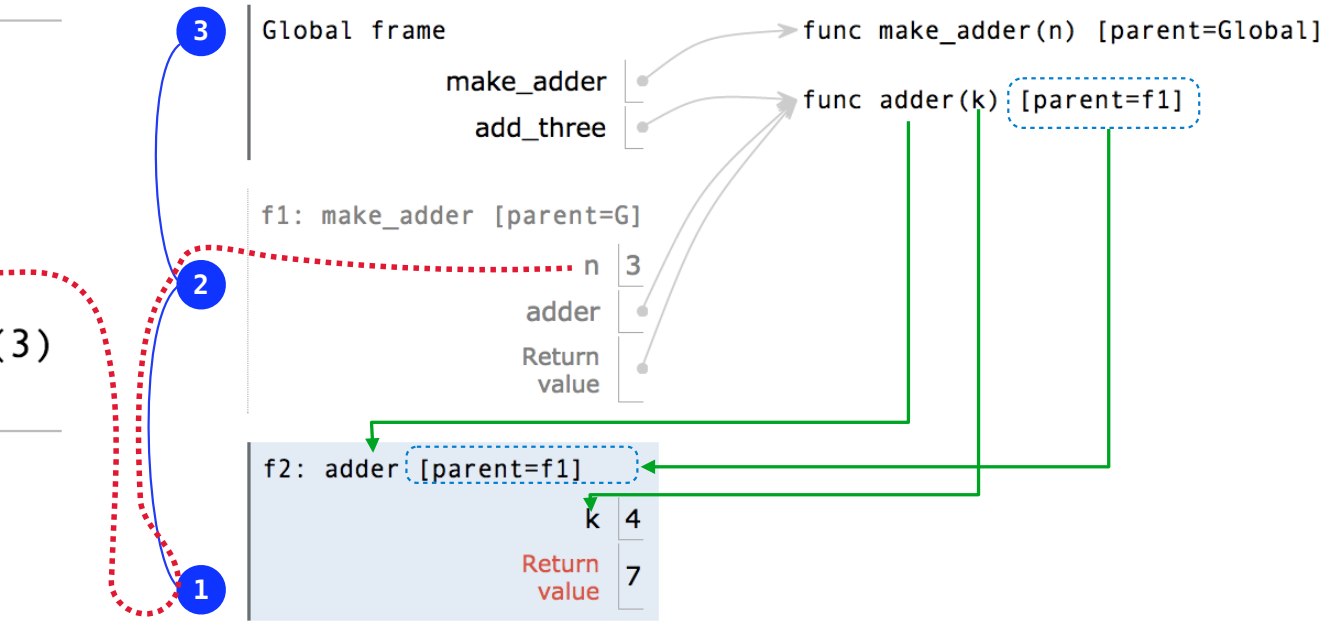

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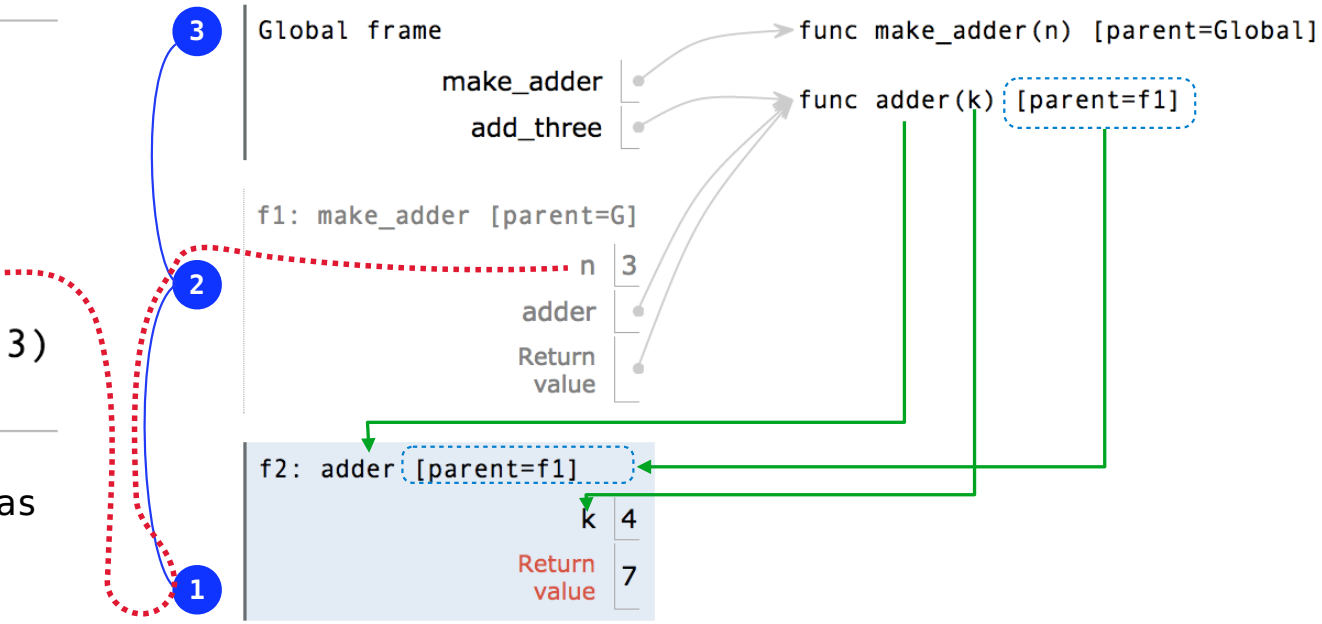

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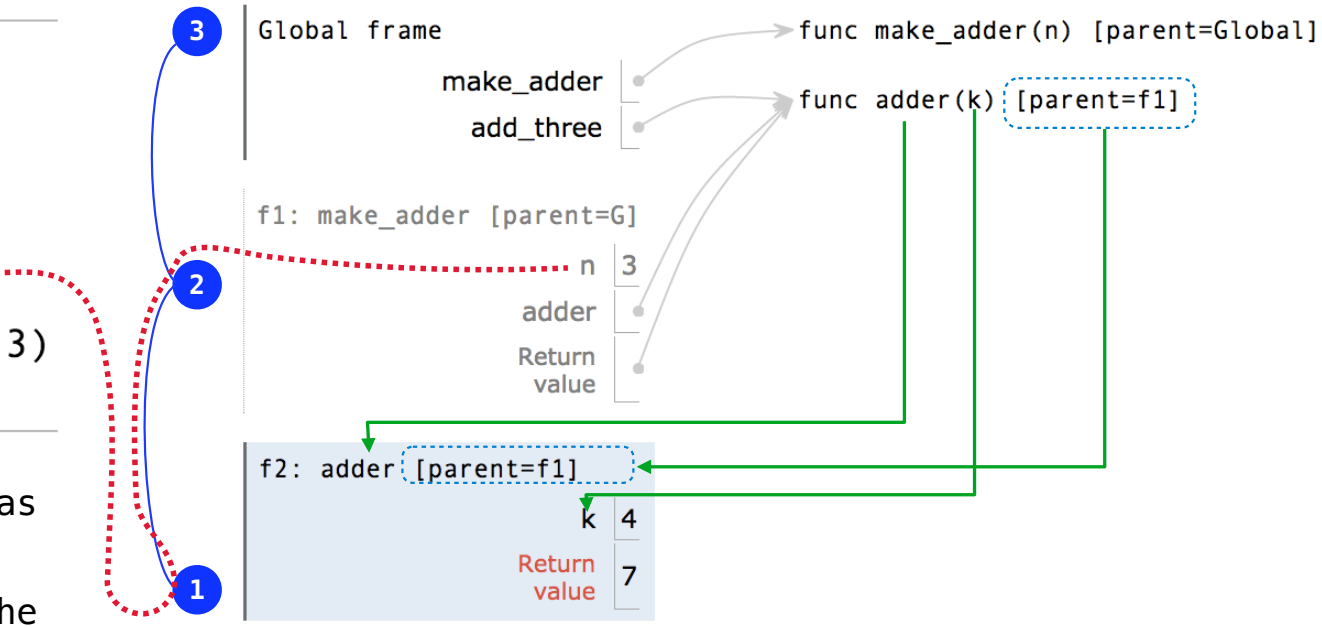

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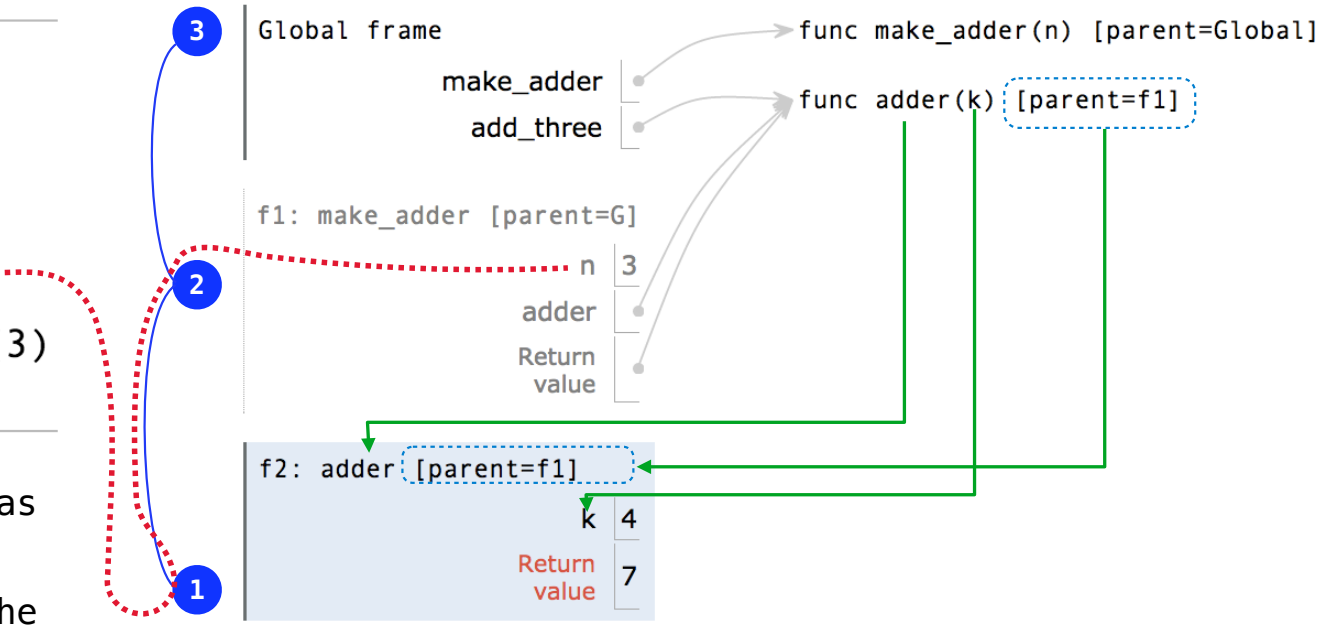

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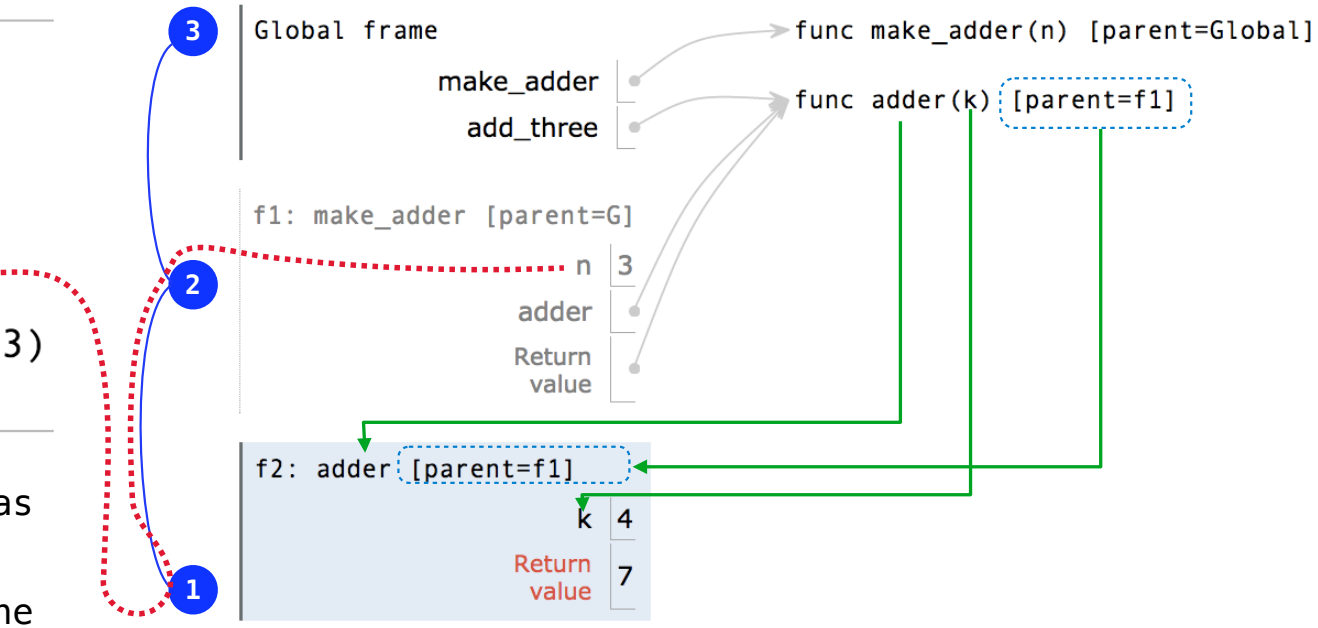

- Every user-defined function has a parent frame (often global)
- The parent of a function is the frame in which it was defined
- Every local frame has a parent frame (often global)
- The parent of a frame is the parent of the function called

http://pythontutor.com/composingprograms.html#code=def%20make_adder%28n%29%3A%0A%20%20%20%20def%20adder%28k%29%3A%0A%20%20%20%20%20%20%20%20return%20k%20%2B%20n%0A%20%20%20%20return%20adder%0A%20%20%20%20%0Athree_more_than%20%3D%20make_adder%283%29%0Aresult%20%3D%20three_more_than%284%29&cumulative=false&curInstr=0&mode=display&origin=composingprograms.js&py=3&rawInputLstJSON=%5B%5D

How to Draw an Environment Diagram

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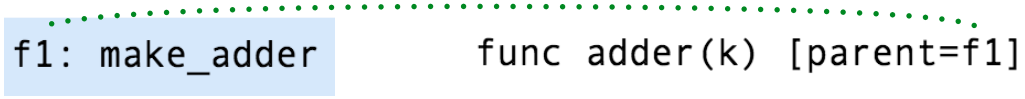

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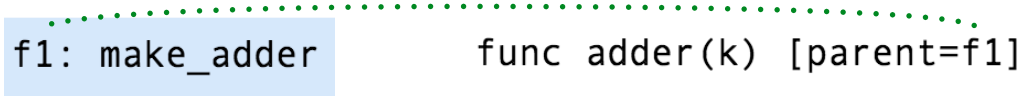

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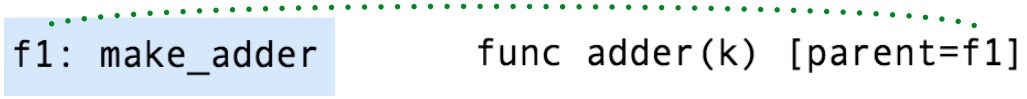

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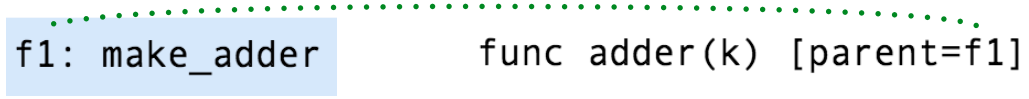

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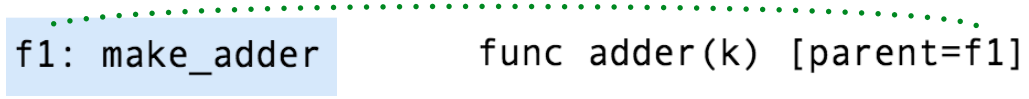

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When a function is called:

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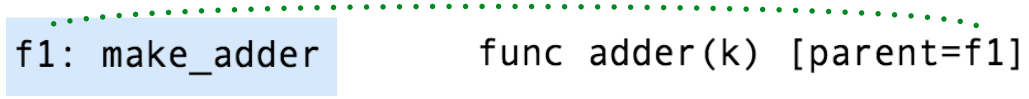

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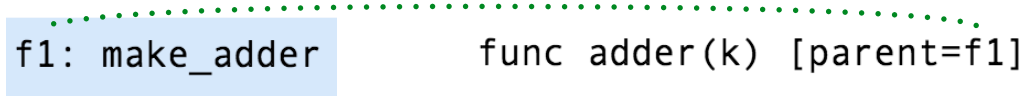

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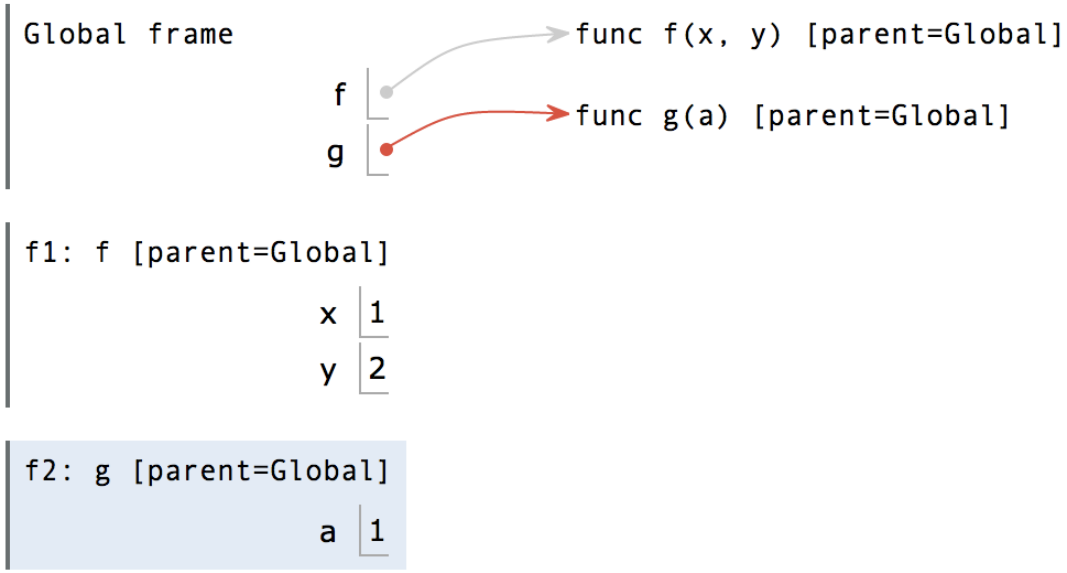
3. Bind the `<formal parameters>` to the arguments in the local frame.
4. Execute the body of the function in the environment that starts with the local frame.

Local Names

(Demo)

Local Names are not Visible to Other (Non-Nested) Functions

```
1  def f(x, y):
2      return g(x)
3
4  def g(a):
5      return a + y
6
7  result = f(1, 2)
```



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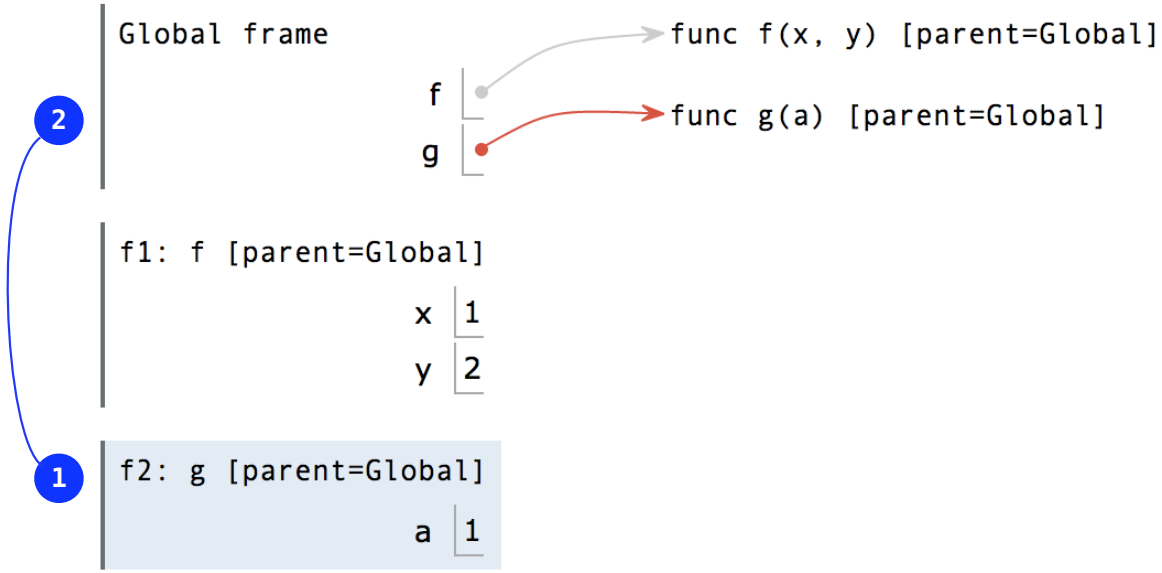

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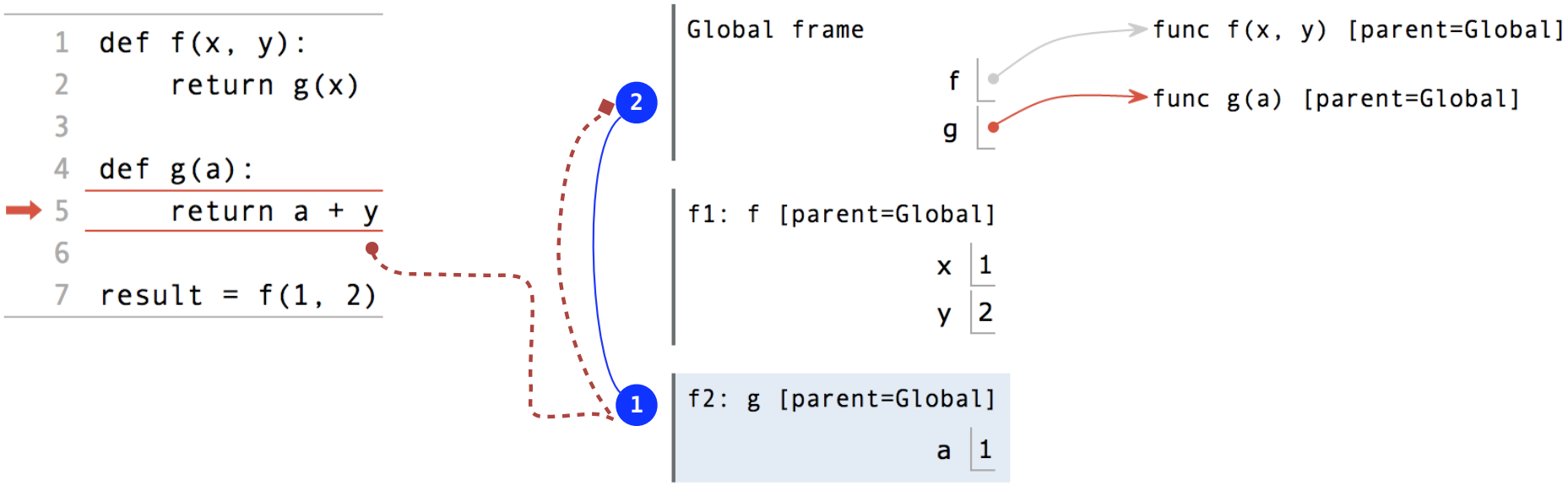

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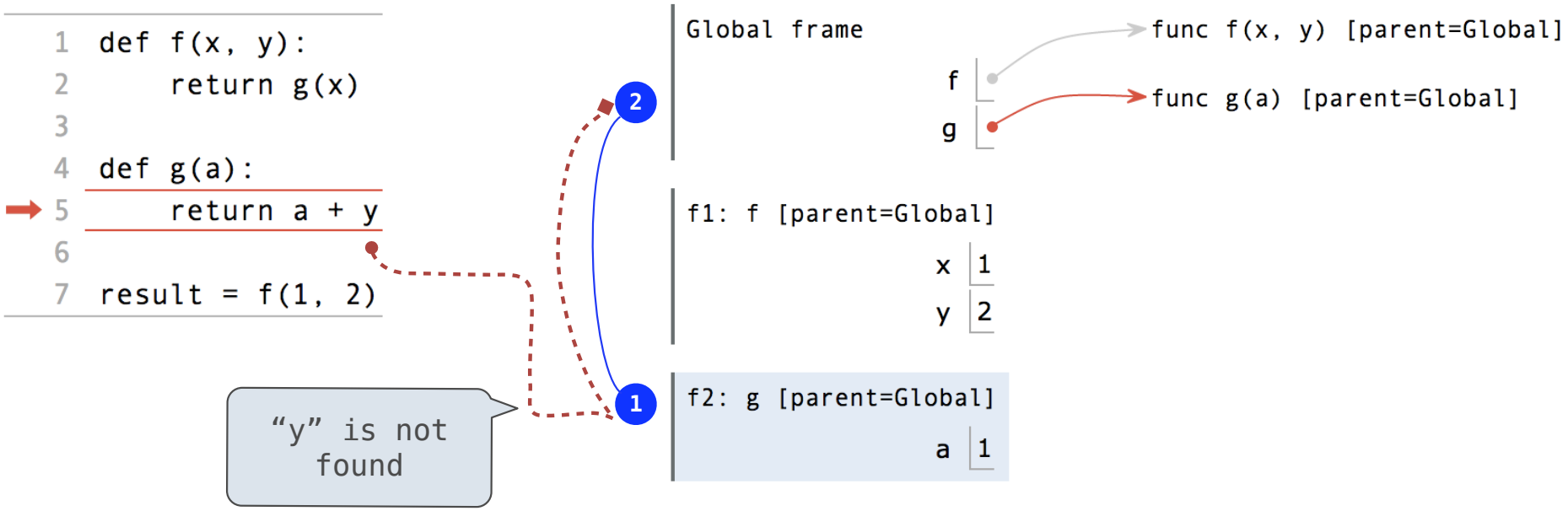

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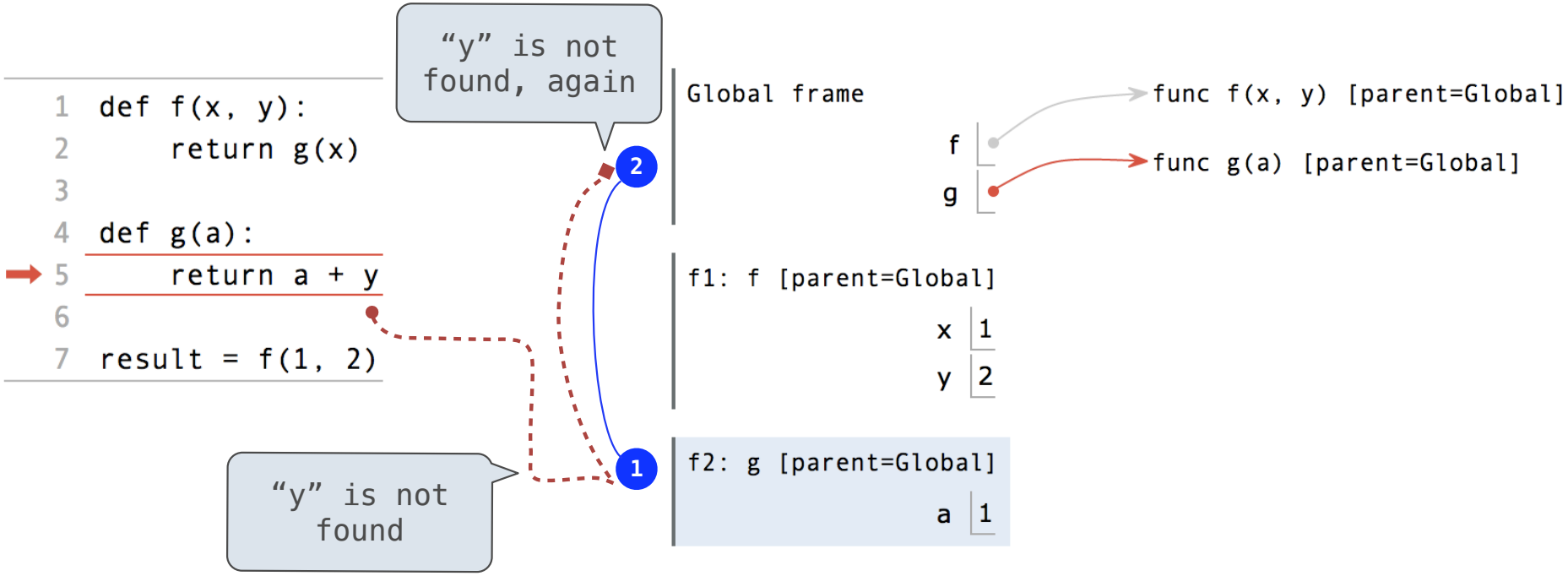

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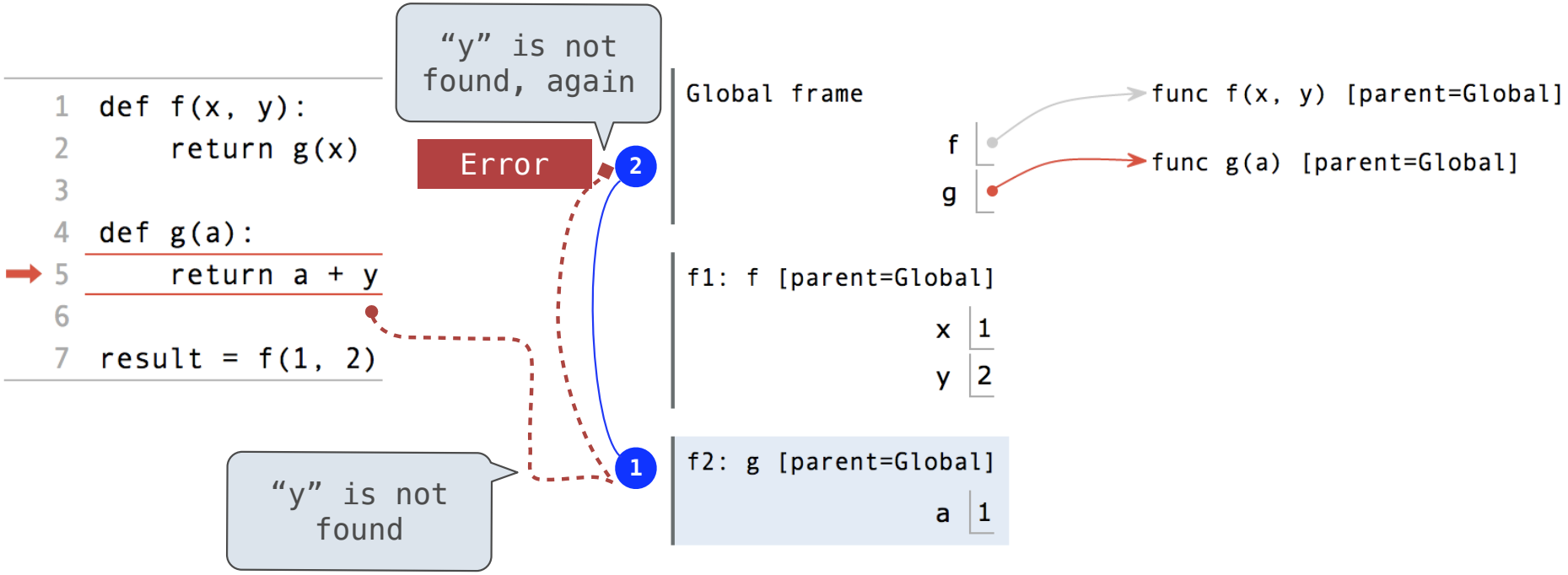

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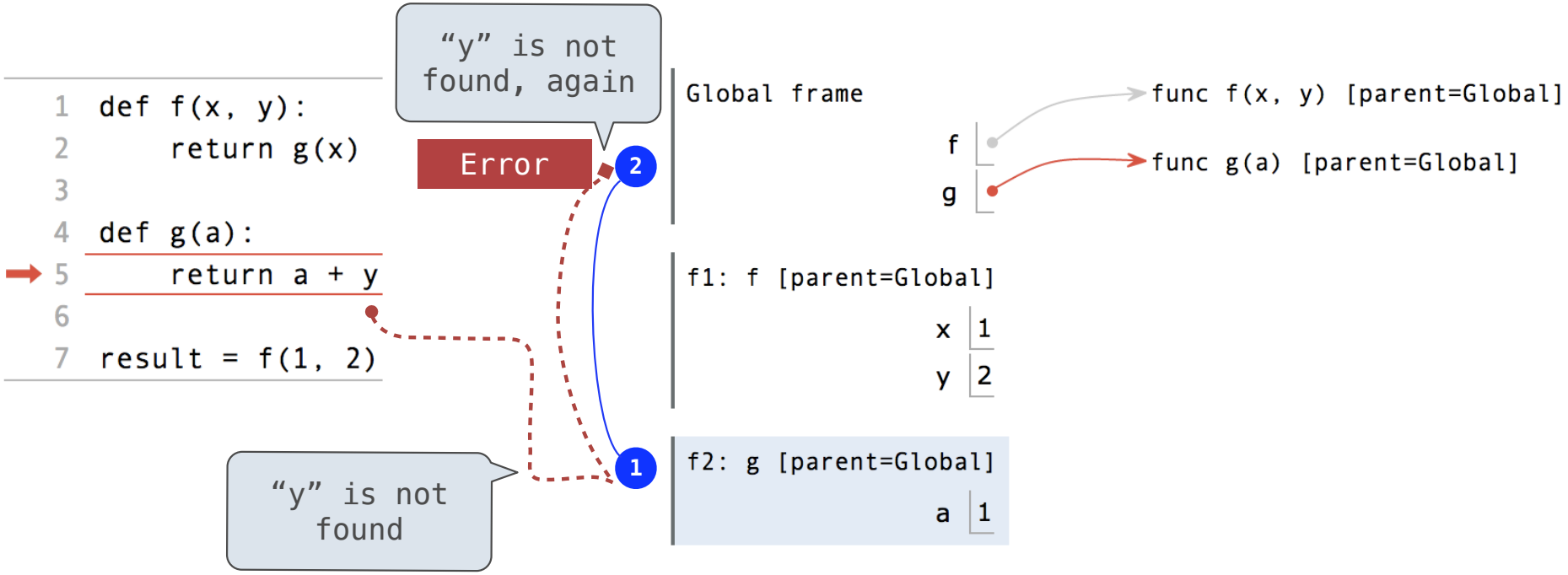

- An environment is a sequence of frames.

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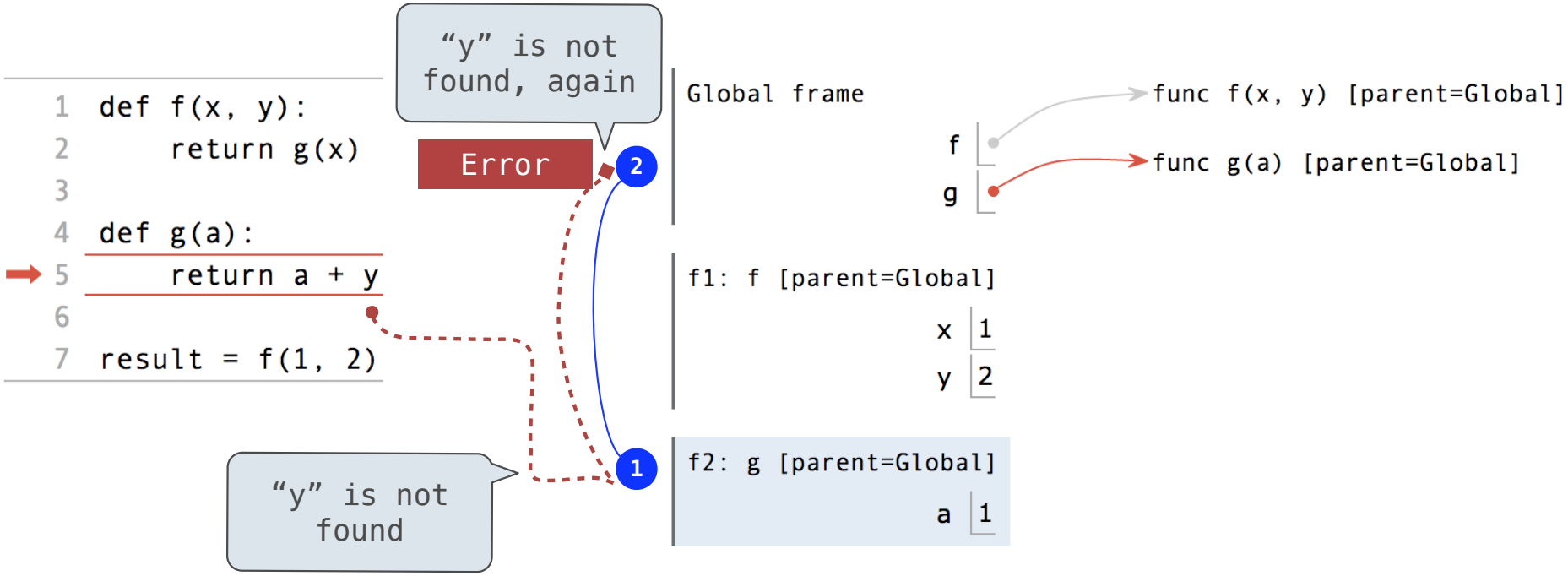

- An environment is a sequence of frames.
- The environment created by calling a top-level function (no def within def) consists of one local frame, followed by the global frame.

http://pythontutor.com/composingprograms.html#code=def%20f%28x,
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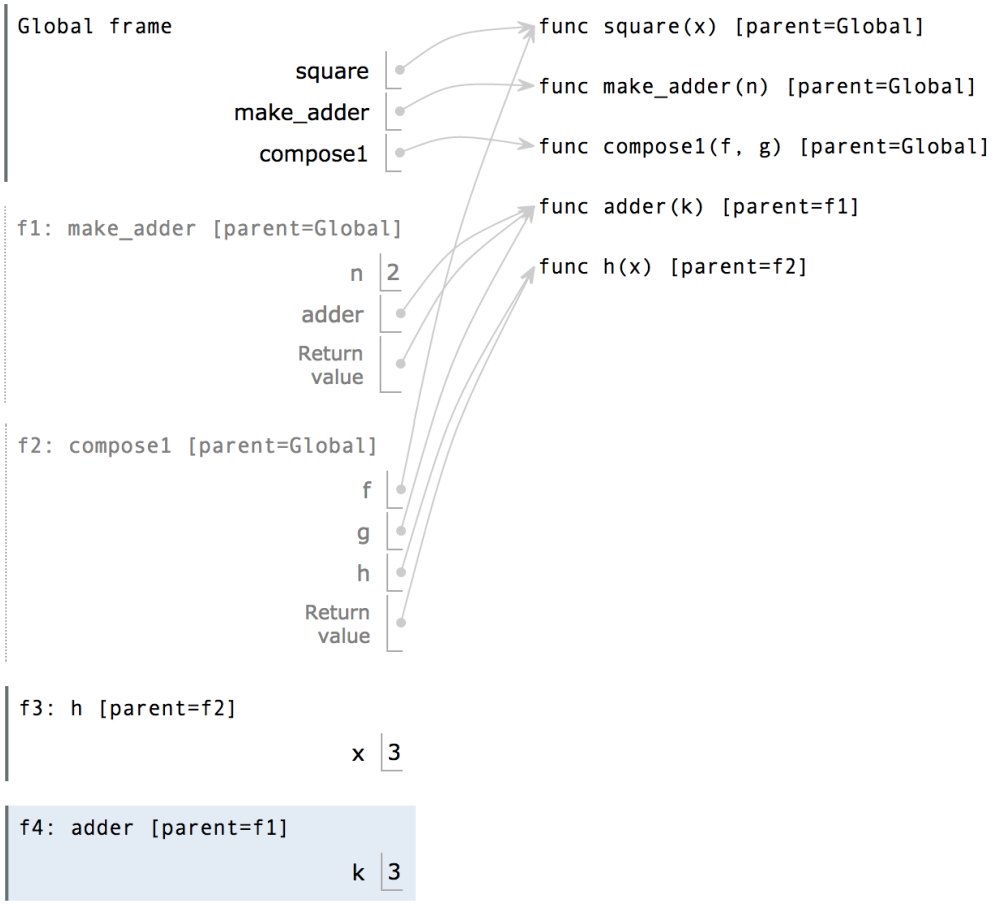
Function Composition

(Demo)

The Environment Diagram for Function Composition

```
1  def square(x):
2      return x * x
3
4  def make_adder(n):
5      def adder(k):
6          return k + n
7      return adder
8
9  def compose1(f, g):
10     def h(x):
11         return f(g(x))
12     return h
13
14 compose1(square, make_adder(2))(3)
```



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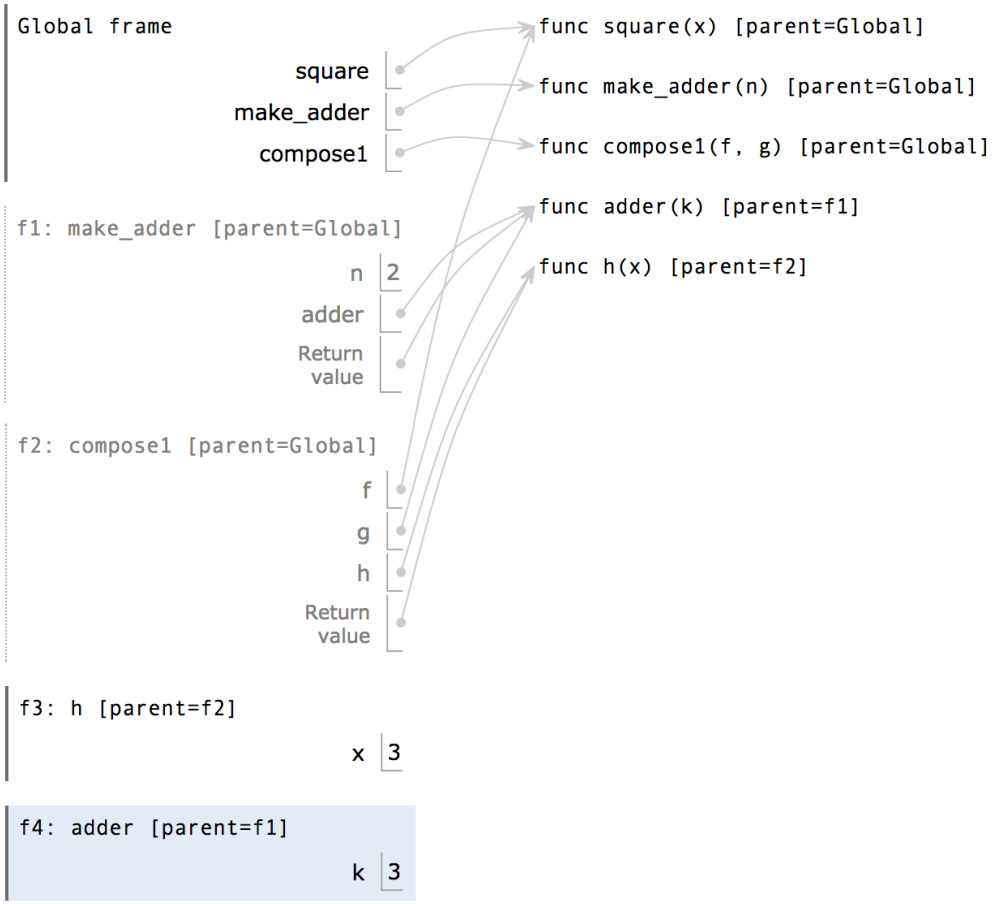

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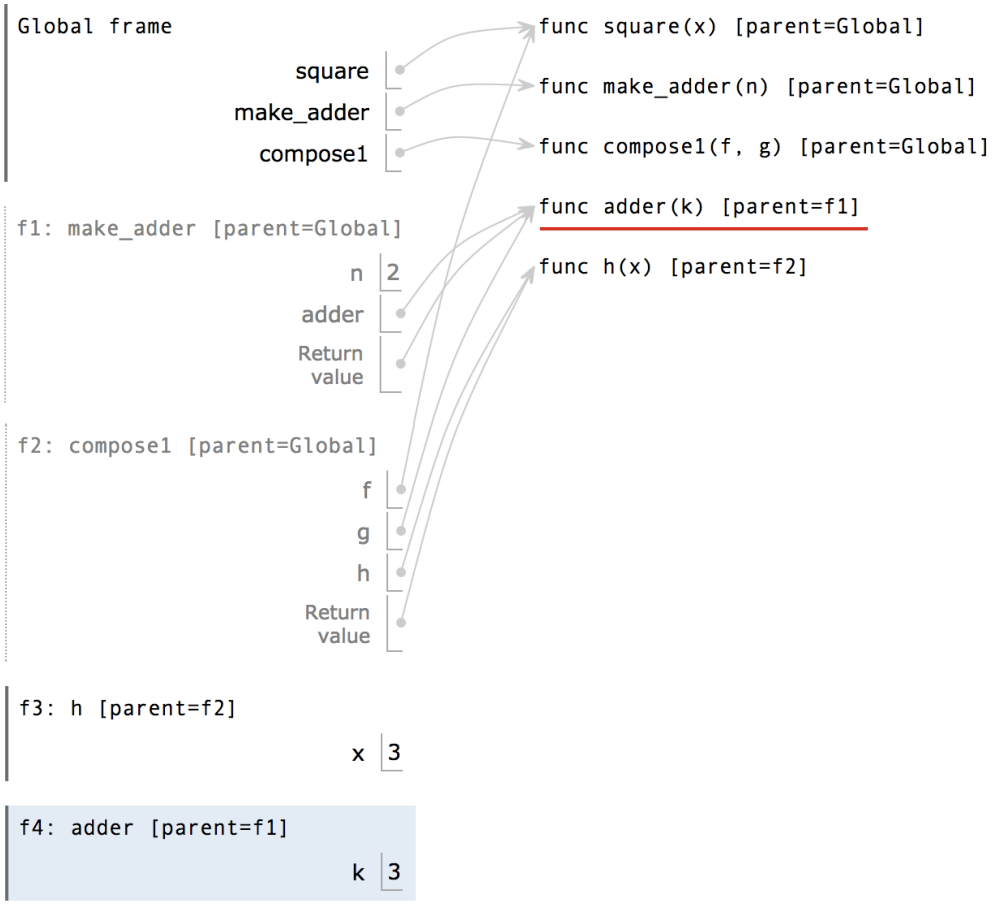

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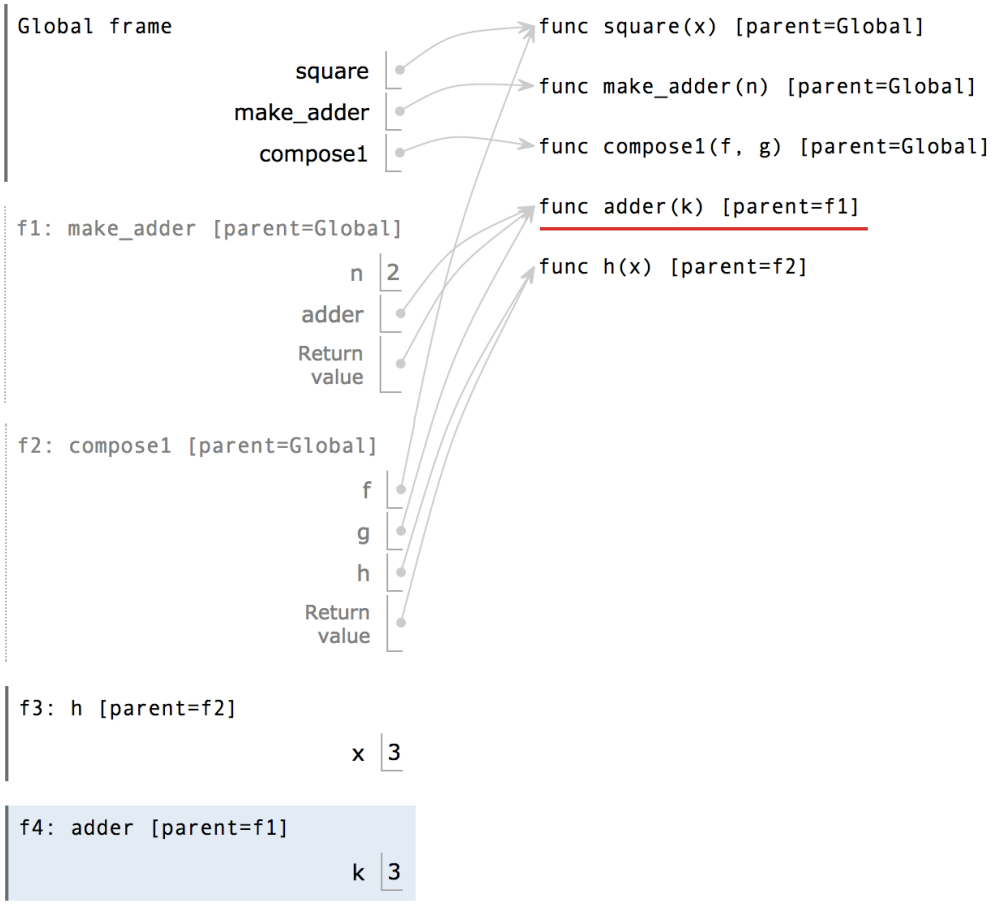

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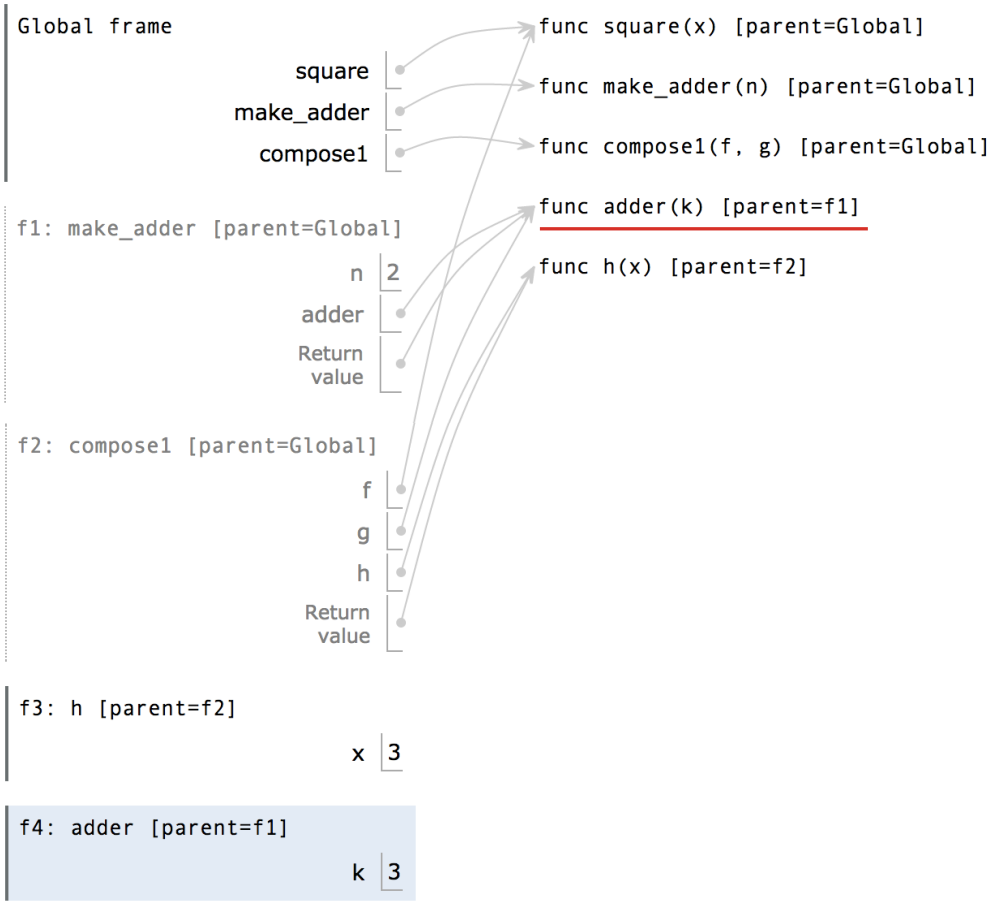

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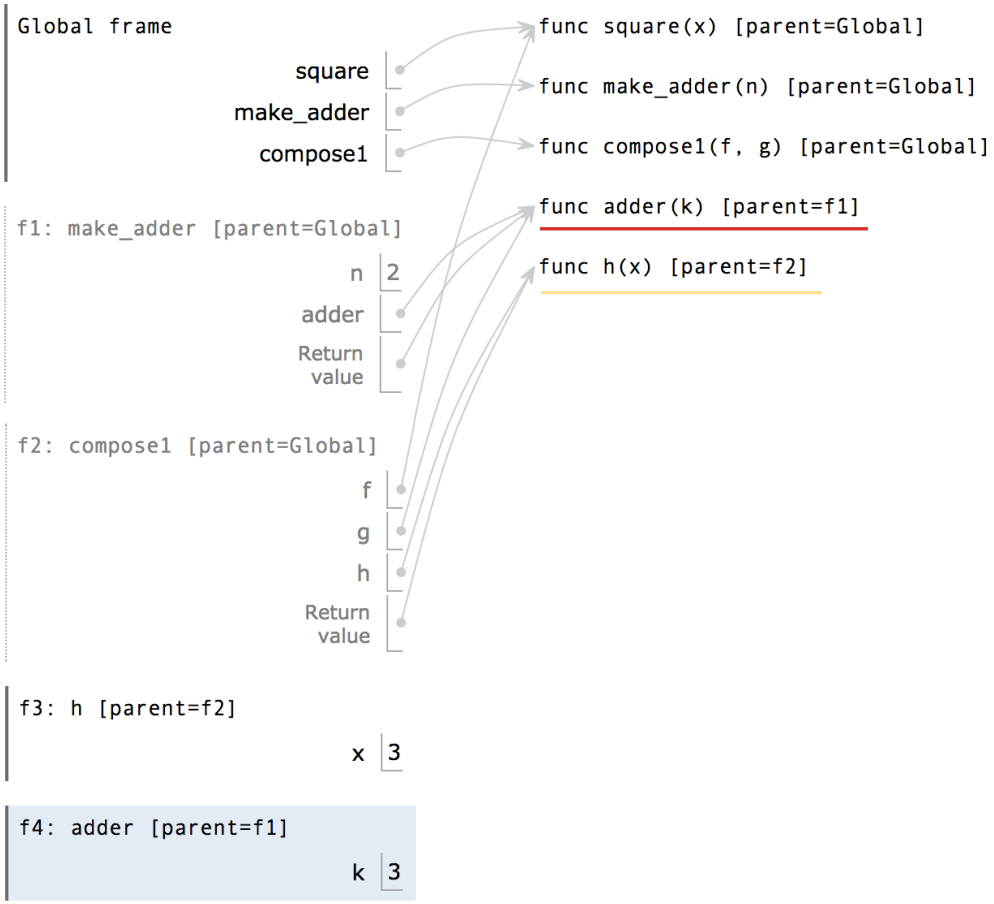

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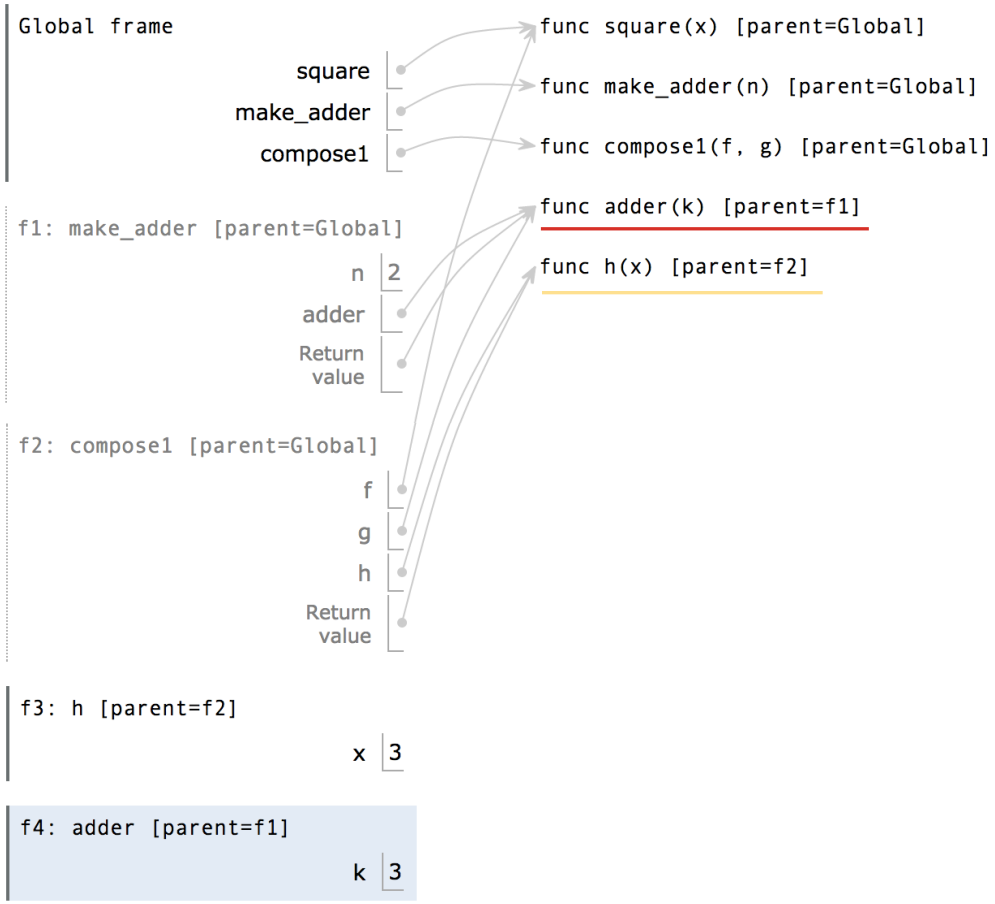

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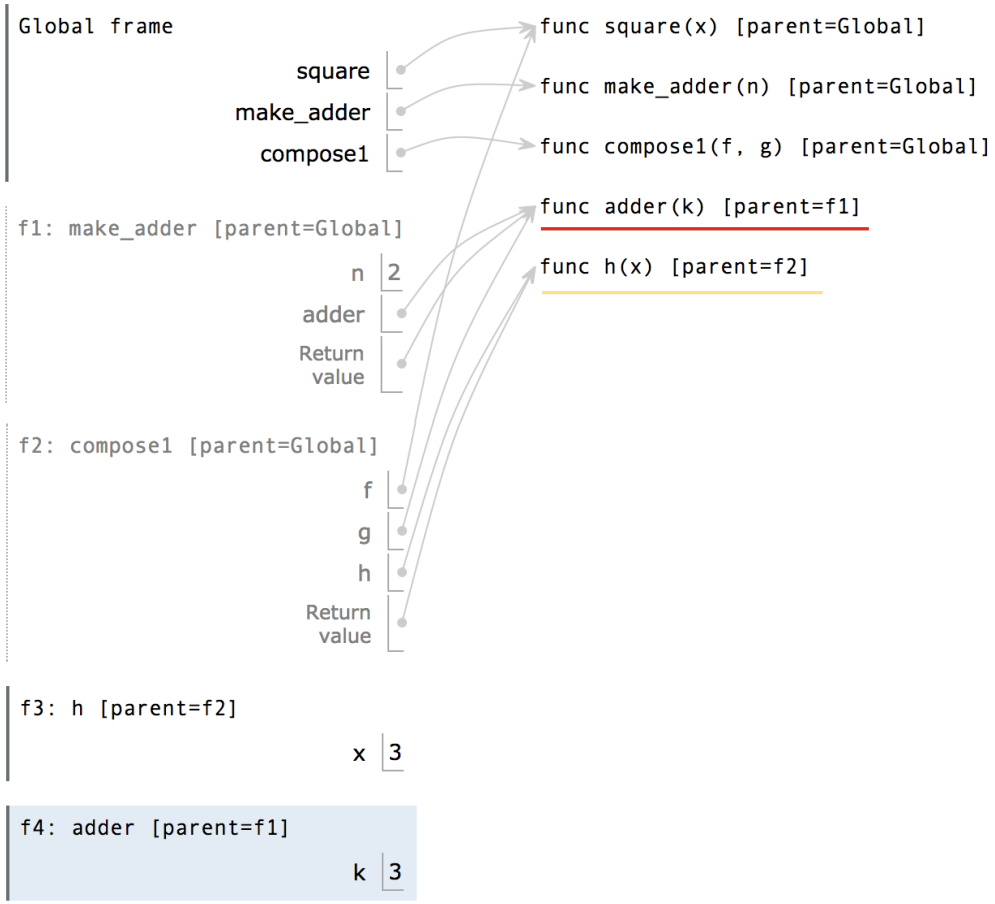

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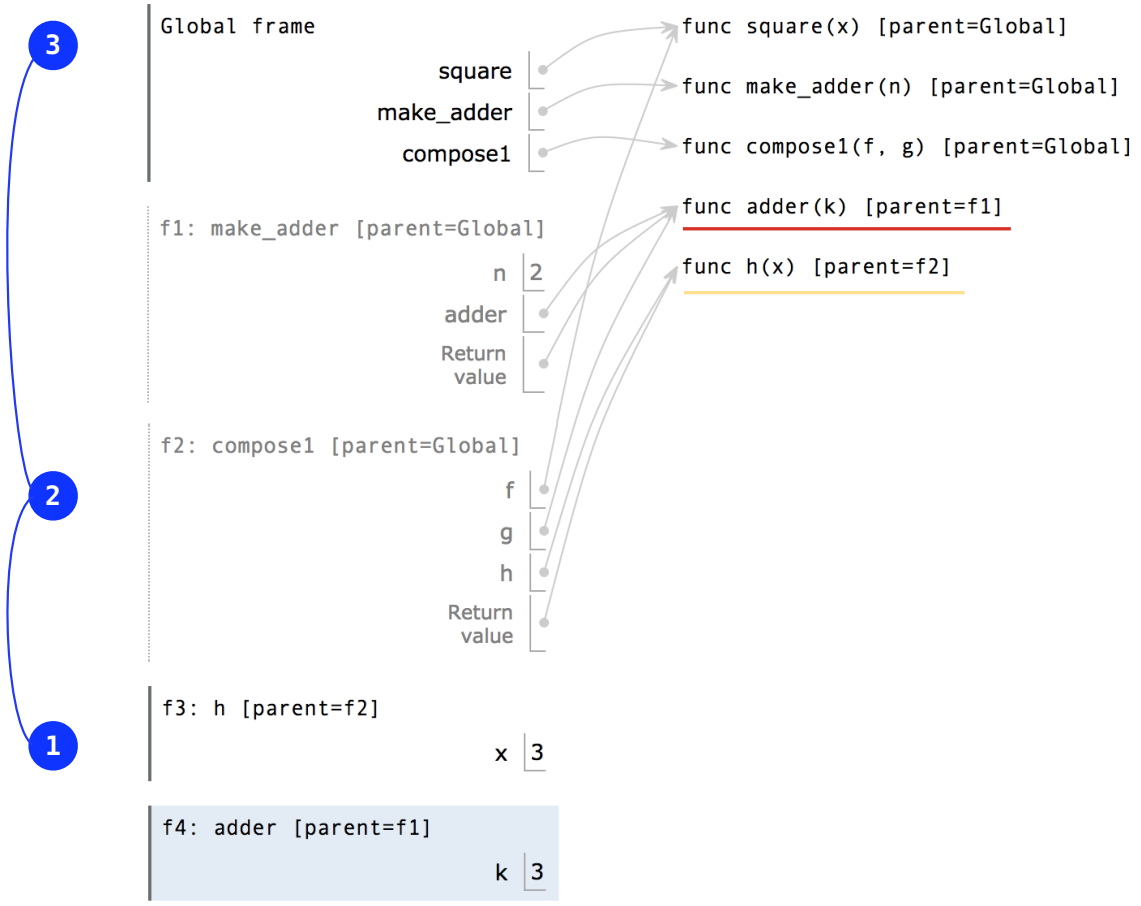

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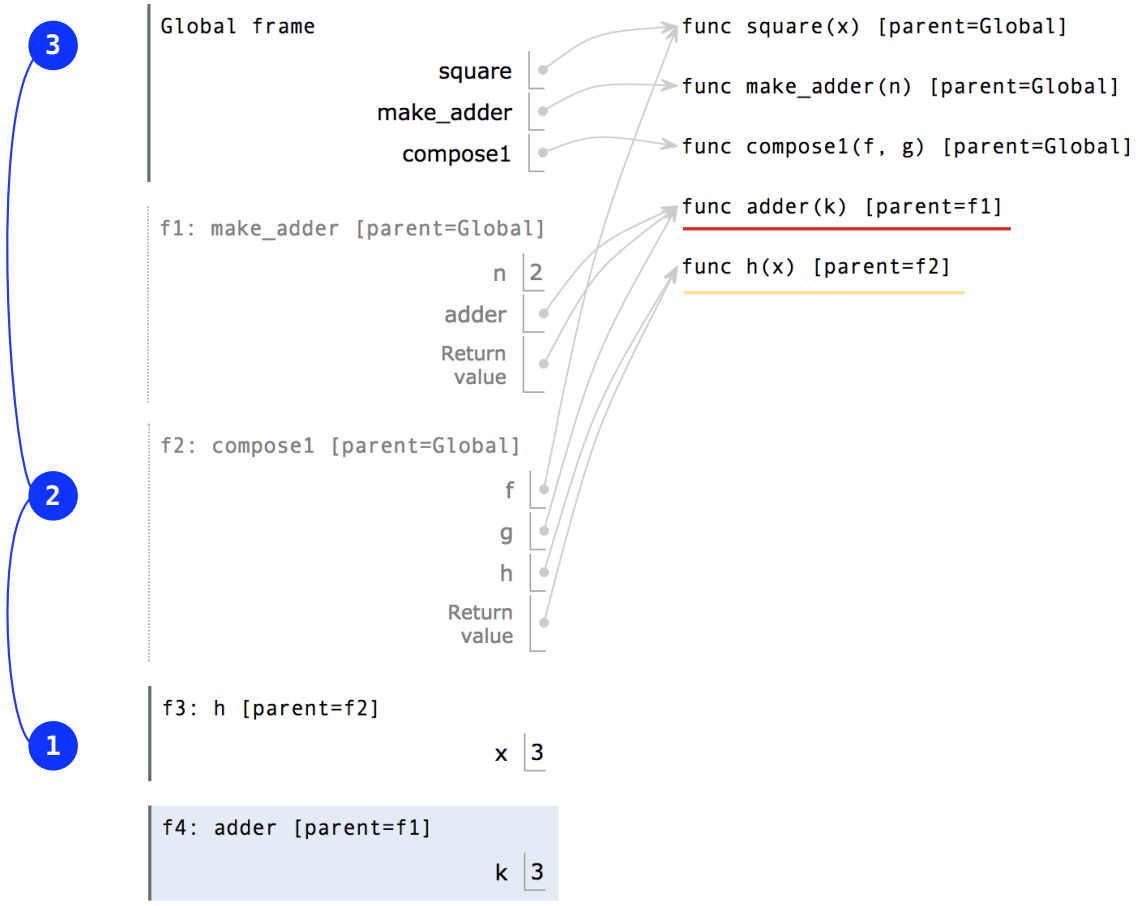

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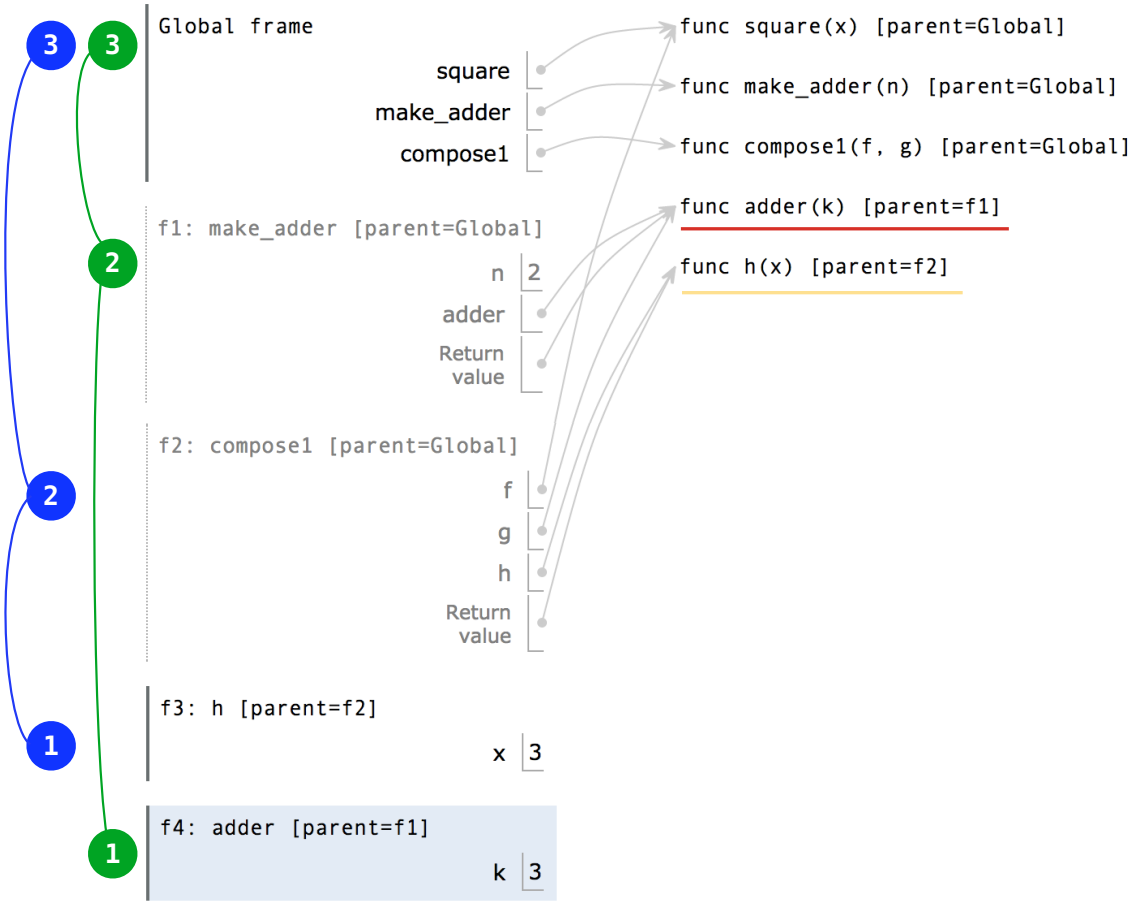

Lambda Expressions

(Demo)

Lambda Expressions

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>>> x = 10
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A function

with formal parameter x

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Must be a single expression

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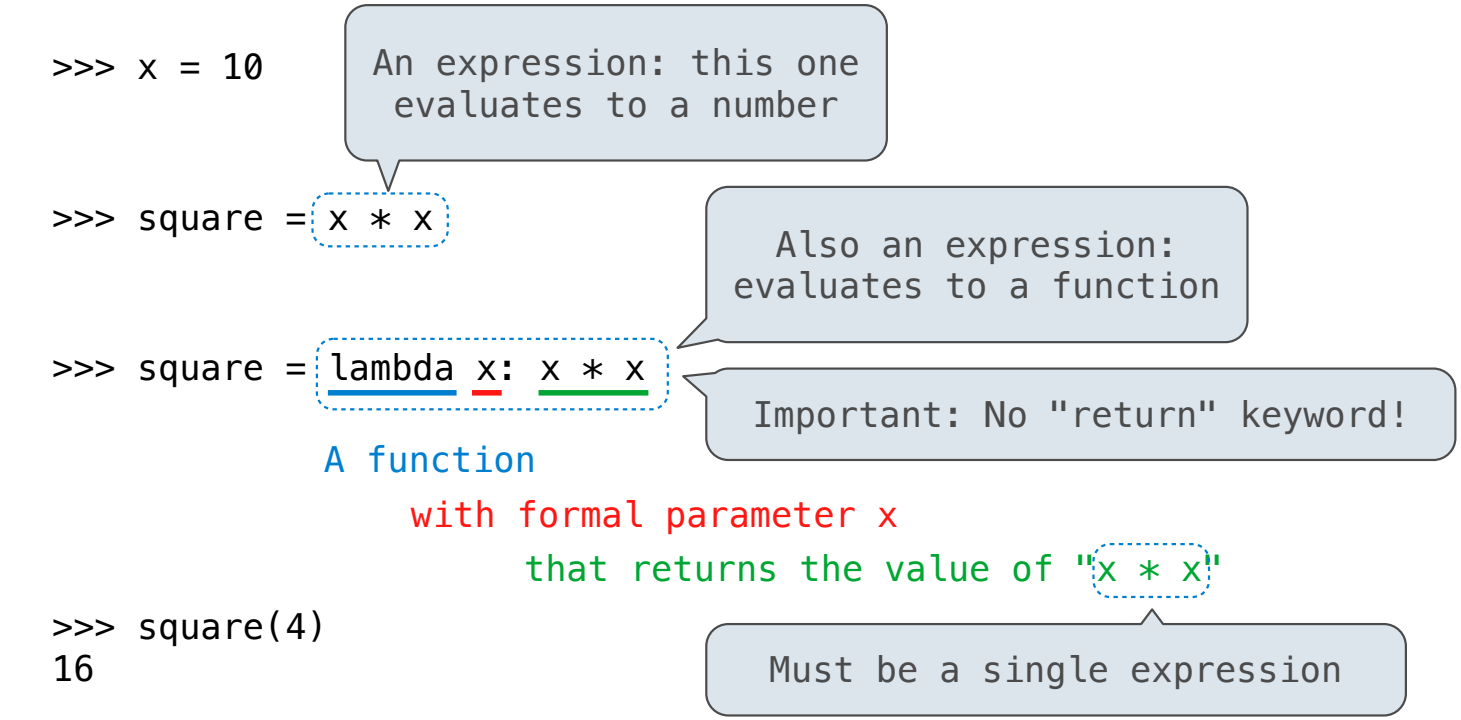
Lambda expressions are not common in Python, but important in general

Lambda Expressions



Lambda expressions are not common in Python, but important in general

Lambda expressions in Python cannot contain statements at all!

Lambda Expressions Versus Def Statements

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VS

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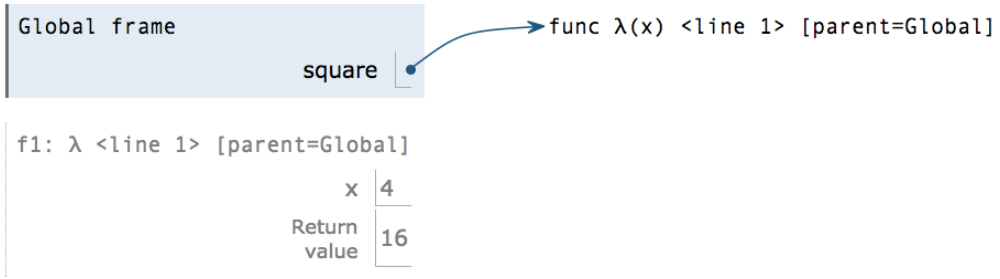

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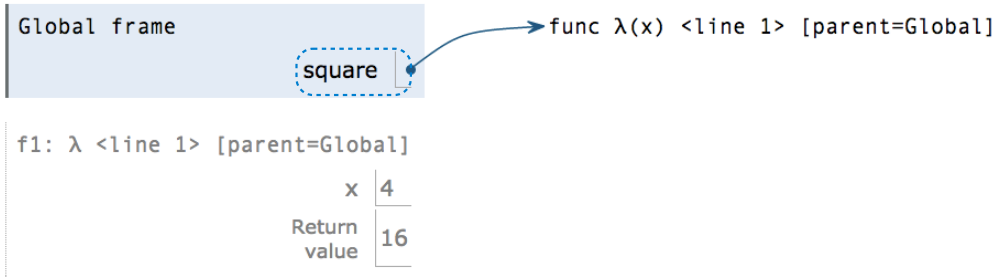

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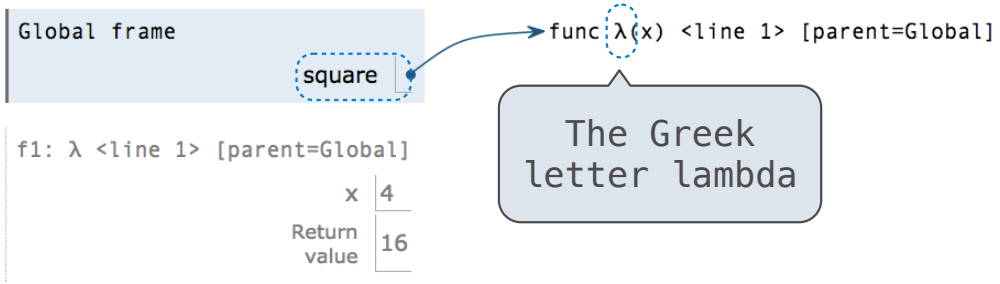

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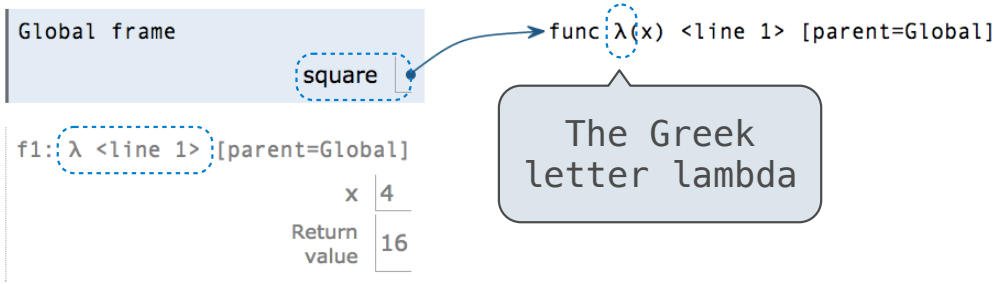

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