

Higher-Order Functions

Announcements

Example: Prime Factorization

Prime Factorization

Each positive integer n has a set of prime factors: primes whose product is n

...
 $8 = 2 * 2 * 2$
 $9 = 3 * 3$
 $10 = 2 * 5$
 $11 = 11$
 $12 = 2 * 2 * 3$
...

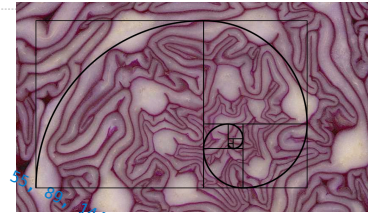
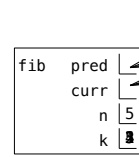
One approach: Find the smallest prime factor of n , then divide by it

$$858 = 2 * 429 = 2 * 3 * 143 = 2 * 3 * 11 * 13$$

(Demo)

Example: Iteration

The Fibonacci Sequence

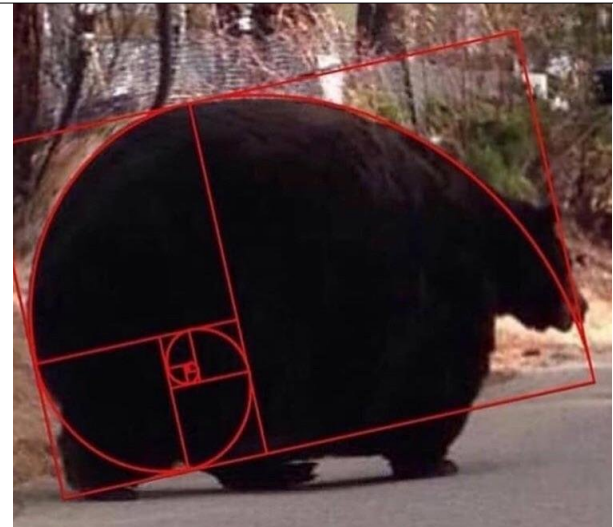


```
def fib(n):  
    """Compute the nth Fibonacci number, for N >= 1."""  
    pred, curr = 0, 1 # 0th and 1st Fibonacci numbers  
    k = 1 # curr is the kth Fibonacci number  
    while k < n:  
        pred, curr = curr, pred + curr  
        k = k + 1  
    return curr
```

The next Fibonacci number is the sum of the current one and its predecessor



Go Bears!



Control

Boolean Contexts

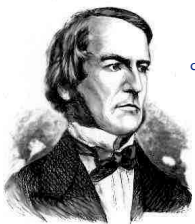


George Boole

```
def absolute_value(x):  
    """Return the absolute value of x."""  
    if x < 0:  
        return -x  
    elif x == 0:  
        return 0  
    else:  
        return x
```

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Boolean Contexts



George Boole

```
def absolute_value(x):  
    """Return the absolute value of x."""  
    if x < 0:  
        return -x  
    elif x == 0:  
        return 0  
    else:  
        return x
```

Two boolean contexts

False values in Python: False, 0, '', None (more to come)

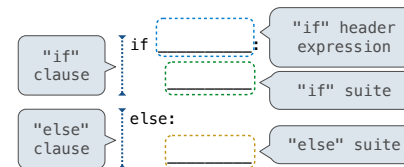
True values in Python: Anything else (True)

(Demo)

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If Statements and Call Expressions

Let's try to write a function that does the same thing as an if statement.



Execution Rule for Conditional Statements:

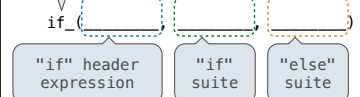
Each clause is considered in order.

1. Evaluate the header's expression (if present).
2. If it is a true value (or an else header), execute the suite & skip the remaining clauses.

(Demo)

```
def if_(c, t, f):  
    if c:  
        return t  
    else:  
        return f
```

This function doesn't exist



Evaluation Rule for Call Expressions:

1. Evaluate the operator and then the operand subexpressions

2. Apply the function that is the value of the operator to the arguments that are the values of the operands

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Short-Circuiting Expressions

Logical Operators

To evaluate the expression **<left> and <right>**:

1. Evaluate the subexpression **<left>**.
2. If the result is a false value **v**, then the expression evaluates to **v**.
3. Otherwise, the expression evaluates to the value of the subexpression **<right>**.

To evaluate the expression **<left> or <right>**:

1. Evaluate the subexpression **<left>**.
2. If the result is a true value **v**, then the expression evaluates to **v**.
3. Otherwise, the expression evaluates to the value of the subexpression **<right>**.

(Demo)

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Higher-Order Functions

Generalizing Over Computational Processes

The common structure among functions may be a computational process, rather than a number.

$$\sum_{k=1}^5 k = 1 + 2 + 3 + 4 + 5 = 15$$

$$\sum_{k=1}^5 k^3 = 1^3 + 2^3 + 3^3 + 4^3 + 5^3 = 225$$

$$\sum_{k=1}^5 \frac{8}{(4k-3) \cdot (4k-1)} = \frac{8}{3} + \frac{8}{35} + \frac{8}{99} + \frac{8}{195} + \frac{8}{323} = 3.04$$

(Demo)

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Summation Example

```
def cube(k):  
    return pow(k, 3)
```

Function of a single argument
(not called "term")

```
def summation(n, term):  
    """Sum the first n terms of a sequence.
```

A formal parameter that will
be bound to a function

```
>>> summation(5, cube)
```

```
225
```

The cube function is passed
as an argument value

```
    total, k = 0, 1  
    while k <= n:  
        total, k = total + term(k), k + 1  
    return total
```

The function bound to term
gets called here

```
0 + 1 + 8 + 27 + 64 + 125
```